Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 254 SEPTEMBER, 1952 No. 3

SOME EVENTS IN THE EARLY HISTORY OF THE OSCILLATING MAGNETRON

BY

W. C. WHITE[1]

Major E. S. Darlington, who is with the U. S. Far East Air Forces in Japan, has spent considerable of his "off" time looking up Dr. Hidetsugu Yagi and discussing with him the early work in Japan on the oscillating magnetron. Before quoting from Major Darlington's interesting letters, it is best to review briefly the beginnings of this device. Many articles have been written on the magnetron and some include historical material (1, 2, 3, 4).[2]

The word "magnetron" was used first by Dr. A. W. Hull (5) to describe a tube having a cylindrical anode with a concentric thermionic filamentary cathode. In this tube, the paths of the electrons were controlled by varying the magnetic field, usually accomplished by a change in the flow of current in an electromagnetic solenoid surrounding the tube. Thus, it was a sort of electromagnetic counterpart of the voltage controlled triode.

The two earliest references on the obtaining of oscillations from a tube having a constant magnetic field appeared in 1924. One was by Dr. A. Zacek from the University of Prague, Czechoslovakia, which appeared in a publication in that country (6). Subsequent to Dr. Yagi's disclosures, Dr. Zacek published a brief article (7) (in German) on his work in which he mentioned oscillations at about 29 centimeters. The other was the publication of a dissertation by Erich Habann from the University of Jena in Germany (8). Apparently the work of both of these men escaped attention, at least in the U.S.A., for a number of years. Not only was such a device well ahead of its time, but radio broadcasting and electronics were flooded with many immediately useful

[1] Research Laboratory, General Electric Company, Schenectady, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

tube developments and their applications, and these were being rapidly put into commercial use.

Dr. Habann devoted most of his article to the results of his study of the relative division of electron current from a single thermionic cathode among a group of anodes in a tube inside of an electromagnet. Such a tube is shown in Fig. 1, which is copied from his paper. However, he

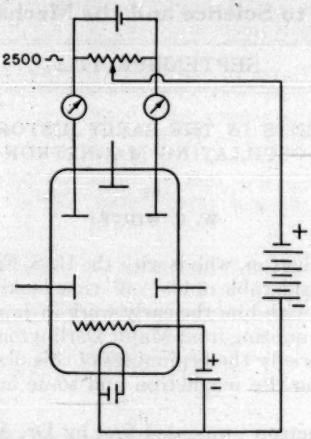

FIG. 1. A sketch of one of Habann's experimental tubes and its circuit to test the division of current among anodes in a magnetic field. (Copied from Habann's article.)

did find some definite negative resistance relationships and there is a somewhat vague reference to a high-frequency energy output of 11 watts, but details are lacking and no frequency is mentioned. There is one figure in his article, a copy of which is shown in Fig. 2, that illus-

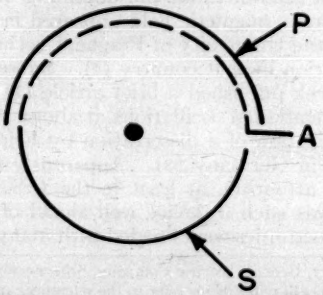

FIG. 2. Another figure from the Habann article which depicts a split-cylinder type of anode plus a grid. He found a negative resistance characteristic and produced oscillations.

trates a split-cylinder-anode form of construction incorporating a third electrode in the form of a grid. This article indicates that its author was cognizant of the spiral paths of electrons under the combined influence of electrostatic and electromagnetic fields.

After a gap of some three years, the next event occurred on almost the opposite side of the world—a 1927 publication (in English) by Professor Kinjiro Okabe of the Imperial University, Sendai, Japan (9). This also seems to have escaped attention in this country.

Dr. Yagi of the College of Engineering, Tohoku Imperial University, Sendai, Japan, brought the first news of the high-frequency, multisection anode, oscillating magnetron to this country in 1928. Major Darlington's letters tell the background of this event as follows:

> How did Dr. Hidetsugu Yagi first learn about the existence of Dr. A. W. Hull's magnetron and how did he become interested in the magnetron and then develop the first microwave power magnetron which today is the most important component of radar and similar electronic devices?
>
> I learned the answer to the above questions directly from Dr. Yagi himself in Tokyo, Japan, on 26 November 1951 at his residence.
>
> I had not seen Dr. Yagi since 1928 when he visited the Research Laboratory of the General Electric Company at Schenectady, New York. Although he told me that he presently lived in a shabby home and was living in simple Japanese style while going through the post-war conditions in Japan, Dr. Yagi's personality and graciousness entirely overshadowed any thought which might exist in my mind as to living conditions.
>
> As we sat around the charcoal fire in his room, Dr. Yagi first told about his former home and books. In April, 1945, a fire bomb set his house afire and it burned to the ground. All his notes, books and research data covering over 30 years of his intensive work in the field of electronics and physics were completely destroyed. Dr. Yagi mentioned that the house burned down in a few hours—but it took four or five days for the fire to die out, owing to the large charred pile of books in the center of the house slowly and gradually being destroyed forever.
>
> Dr. Yagi is now 65 years old and is currently: Member, Japanese Science Council; Member, Foreign Investment Commission; President, Patent Pool Corporation of Japan; Honorary Professor, Osaka University; and Member, Japan Academy.
>
> It is good to know that Dr. Yagi is playing a vital part in the reconstruction of Japan so that it will again be able to become a great nation and part of the United Nations.
>
> Now as to the magnetron itself. Dr. Yagi was always a keen student and follower of Dr. J. A. Fleming and his work on the diode during the latter's early research work. In 1916, Dr. Yagi left Fleming's laboratory in London and went to Cruft Laboratory at Harvard University, where he further studied Fleming's contributions to the field of electronics.
>
> Upon Dr. Yagi's return to the College of Engineering, Tohoku Imperial University at Sendai, Japan, he instituted a course at the University in the functioning of the diode and added classroom experiments to be undertaken by the students.
>
> Teaching this course under Dr. Yagi's direction was Professor Okabe. Professor Okabe also became much interested in the functioning of the diode and in an investigation of its characteristics, since teaching the subject brought up questions to which no clear-cut answers could be given.
>
> One of the important questions discussed by these two professors was why did not the diode influenced by an axial magnetic field show a sharp cut-off of the plate cur-

rent as was to be theoretically expected at a definite magnetic field strength. This led to the suggestion, noted later in the letter from Professor Okabe, that minute oscillations might possibly be the cause of this lack of sharp plate cut-off.

Sometime prior to 1924, a Japanese Naval Officer returning from an eastern college in the United States had reported to Dr. Yagi that he had heard of a new tube that was called a "magnetron." All that the Japanese Naval Officer could report to Dr. Yagi was that the "magnetron" was a large vacuum bottle, encircled with a large magnetic coil.

This was Dr. Yagi's first knowledge that the name "magnetron" existed and was the introduction of this word into Japan. The Japanese Naval Officer had been referring to Dr. Hull's magnetron, which was described by Dr. Hull in a 1921 publication (5). Dr. Yagi noted, however, that the Hull tube was different in some ways and operated at much lower frequencies.

Again, as indicated in Professor Okabe's letter quoted later, the oscillations produced with the 'unsplit' anode were very feeble which led to the construction of tubes with special forms of electrodes.

Thus, the microwave oscillating power magnetron was born and was later to be used as the heart of one of World War II's greatest secrets and developments—all thanks to a great scientist who started with a laboratory child of Dr. A. W. Hull and developed that child into a man of power for use in peace and war.

Major Darlington also made inquiries relative to Professor Okabe and found out that he is now on the staff of the Department of Physics, Osaka University, at Osaka. As this is several hundred miles from Tokyo, Major Darlington was unable to visit him, but did correspond with him. Professor Okabe wrote in English as follows:

From 1922 to 1925, I was a lecturer at Tohoku University and was Assistant Professor of the same university from 1925 to 1929. In the beginning of 1927, I and my students obtained an abnormal curve, as shown by dotted line in the figure [Fig. 3], with a magnetron which was manufactured in my laboratory for the purpose of

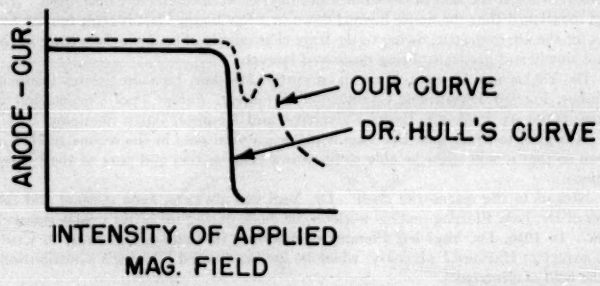

FIG. 3. Copied from a letter from Professor Okabe indicating the discontinuity in the characteristic curve which furnished the clue that the tube was oscillating.

measuring the value of e/m. Then I repeated the same experiment carefully in order to know the cause for this phenomenon and found that the anode current varied slightly whenever I approached my hand to the circuit. Thus, I found a feeble oscillation of about 60 cm in wavelength. Thus, I began my research on magnetron oscillations at ultrashort wavelengths. This first work was published [9].

"As the oscillations obtained with single-anode magnetrons [Dr. Hull's type magnetrons] were weak in intensity, I tried to increase the output power, as well as

to shorten the wavelength, by improving the electrode configuration and invented the split-anode magnetron after experiments with more than 50 magnetrons of different electrode configuration. All of these tubes were manufactured by myself or by my two young assistants.

The above invention was patented in Japan, U.S.A. and Canada. I assigned the U.S.A. and Canada patents to G.E. Company [Schenectady].

The first one who took interest in my work was Dr. Yagi. He gave me valuable suggestions in the course of the development of my invention. In 1928, Dr. Yagi visited America and described my work on magnetron oscillations as well as his work on antennas at a few meetings of electrical engineers in America. He also described my work, as well as his work, on antennas in the 1928 *Proceedings of I.R.E.*

In 1937 I published a small book titled "Magnetron Oscillations of Ultra-Short Wavelengths" [written in English]. In this book, the history of magnetron oscillations is briefly described.

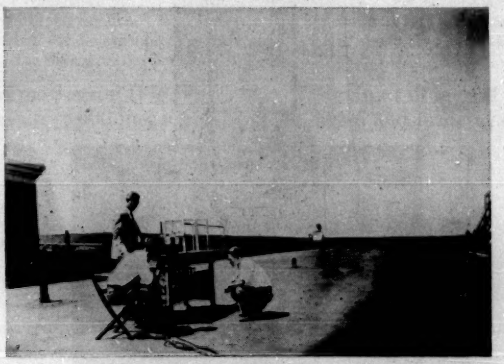

FIG. 4. This picture, taken in 1928, shows a 400-mc. experimental equipment built and operated in Schenectady. Its magnetron transmitting tube and antenna structure were based on information in Dr. Yagi's 1928 article in the *Proc. I. R. E.* Major Darlington, who took part in these tests, is kneeling to the right of the transmitter.

In another letter from Professor Okabe (this time translated from his Japanese handwriting), he refers to Dr. Zacek's work as follows:

> I learned later that Dr. Zacek succeeded in detecting oscillations from a single-anode magnetron before I did. At that time, there was no information available to me on Dr. Zacek's work as it was written in a foreign language. Therefore, I am not the first discoverer of the fact that a magnetron could produce short-wave oscillations. However, I may be the inventor of the multi-split-anode magnetron, and also the first one who succeeded in showing experimentally and theoretically that intense oscillations of very short wavelengths can be produced with magnetrons. The events I have described occupied my time for about ten years. It is gratifying to me to think that with our little help the magnetron has made such progress to its present standing.

The scene now shifts again; this time to the U. S. Dr. Yagi's manuscript, which contained information on the microwave oscillating magnetron, reached the Editor of the *Proceedings of the I.R.E.* on January 30, 1928.

On the evening of February 1 of that year, Dr. Yagi presented his paper before a New York City meeting of the Institute. Subsequently, he repeated this presentation before Institute groups in Washington and Hartford. During this visit to the U. S., Dr. Yagi travelled quite extensively and gave his paper to a group in Schenectady on February 20. His work so impressed the group that the development of such a tube was immediately undertaken. By summer, a complete experimental receiving and transmitting set-up, along the lines suggested by Dr. Yagi's article operating at 400 megacycles, was tested. This equipment is shown in Fig. 4.

FIG. 5. A magnetron transmitting tube for producing frequencies up to 400 megacycles and made available in 1930.

Dr. Yagi's paper (10) was published in the June, 1928, issue of the *Proceedings of the I.R.E.* In the U. S. most of us associate the name of Dr. Yagi with the array of dipoles for forming and directing a beam of microwaves rather than the magnetron and this former device did share with the magnetron the pages of Dr. Yagi's first article. By a rather prophetic coincidence, this same issue of the *Proceedings* contains one of the early articles on a circuit arrangement that was also to become part of the science of radar (11). At the close of the New York City meeting

where Dr. Yagi's paper was first presented, J. H. Dellinger made the following interesting and forward-looking comments:

> Professor Yagi's remarkable work stimulates some thought of a radical order. . . . Radio communication is to a large extent done the wrong way today. And before 1920 radio was all wrong. The only use of radio was for communication between two points, and it was always done by broadcasting in every direction. It was not until 1920 that we had the advent of broadcasting as such, transmission intended for reception by large number of receivers. . . .
>
> It is interesting that 1920 marks not only the rise of broadcasting but also the beginning of directive radio. Ideally, radio transmissions should be broadcast in every direction only when intended for reception in every direction, and should be sent as nearly as possible in one line when intended for reception by one receiver. Since 1920 we have had the gradual and partial evolution of beam systems and other means of confining a communication more or less to the path desired. One instance is the use of a string of relay stations. Now Professor Yagi has shown us that one of the ways to accomplish the directive function is to use a string of absolutely automatic relay stations, viz., the simple devices he calls "directors."

A small magnetron (Fig. 5) capable of an output of several watts at frequencies up to 400 megacycles was developed and made available in

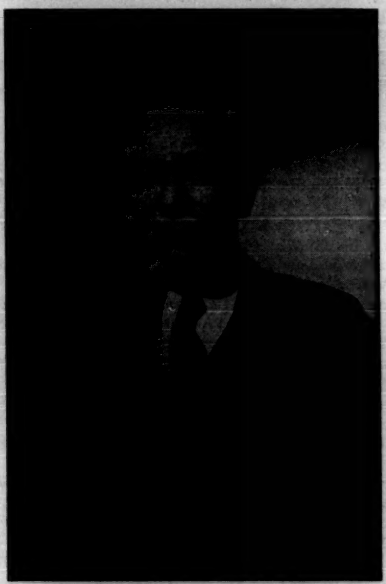

FIG. 6. After taking this picture of Dr. Yagi, Major Darlington drove him to the entrance of the Emperor's palace grounds. At the palace, Dr. Yagi had an appointment with the Emperor and members of the Cabinet for a discussion on plans for Japan's future in the technical field. This explains Dr. Yagi's formal attire.

1930 (12), but there was very little demand for it. Only a few were made and these were used mostly for research work. The oscillating magnetron was little used or needed until shortly before 1940 when the Allies of World War II developed it as an almost ideal generator of microwaves for radar transmitters. There is evidence that the multicavity-resonator type of oscillating magnetron was known prior to 1940 in England, Russia, Germany, and this country (1, 13, 14, 15). Although the Japanese did use the oscillating magnetron for radar during the war (16), they did not develop it to the same extent as the Allies.

Dr. Yagi (Fig. 6) is now actively engaged in advising the new government of Japan along several lines. He writes: "My present activity is rather in economical and political lines, so urgent and important for the rehabilitation of the country."

He is also a consultant for the Japanese government on the problems of formulating television standards for his country. In a recent letter to the author, he states that a new business firm, the Yagi Antenna Co., Ltd., has been formed in Japan and he is its President.

Both Dr. Yagi and Professor Okabe have been cleared by the U. S. military authorities.

REFERENCES

(1) J. T. RANDALL, "The Cavity Magnetron," *Proc. Phys. Soc.*, Vol. 58, p. 247 (1946).
(2) C. G. SUITS, "Physics Today—Engineering Tomorrow," *Electrical Engineering*, March, 1947, p. 241.
(3) G. ROSS KILGORE, "Magnetron as a High-Frequency Generator," *J. Appl. Phys.*, Vol. 8, pp. 666–676 (1937).
(4) ROGER B. COLTON, "Radar in the U. S. Army," *Proc. I. R. E.*, Vol. 33, p. 740 (1945).
(5) A. W. HULL, "The Magnetron," *J. A. I. E. E.*, Vol. 40, p. 715 (1921).
(6) A. ZACEK, "A Method of Generating Short Electromagnetic Waves," *Casopis pro pestorant mathematiky a fysiky* (Prague), Vol. 53, p. 378 (1924); (summary in *Zeits. f. Hochfrequenz*, Vol. 32, p. 172 (1928).
(7) A. ZACEK, "On a Method for Producing Very Short Electromagnetic Waves," *Zeits. f. Hochfrequenz*, Vol. 32, p. 172 (1928).
(8) E. HABANN, "New Generator Tube," *Zeits. f. Hochfrequenz*, Vol. 24, p. 115 (1924).
(9) KINJIRO OKABE, "A New Method for Producing Undamped Extra-Short Electromagnetic Waves, *Proc. Imperial Acad.* (Japan), Vol. 3, No. 4, 1927.
(10) HIDETSUGU YAGI, "Beam Transmission of Ultra-Short Waves," *Proc. I. R. E.*, Vol. 16, pp. 715–741 (1928).
(11) M. A. TUVE AND O. DAHL, "A Transmitter Modulating Device for the Study of the Kennelly-Heaviside Layer by the Echo Method," *Proc. I. R. E.*, Vol. 16, p. 794 (1928).
(12) W. C. WHITE, "Producing Very High Frequencies by Means of the Magnetron," *Electronics*, Vol. 1, p. 34 (1930).
(13) N. F. ALEKSEEV AND D. D. MALAIROV, "Generation of High-Power Oscillations with a Magnetron in the Centimeter Band," *Proc. I. R. E.*, Vol. 32, pp. 136–139 (1944).
(14) U. S. Patent No. 2,270,777.
(15) U. S. Patent No. 2,063,342.
(16) MARVIN HOBBS, "Japanese Magnetrons," *Electronics*, Vol. 19, p. 114 (1946).

SYNTHESIS OF A NONLINEAR DYNAMIC VIBRATION ABSORBER*

BY

ROBERT E. ROBERSON[1]

ABSTRACT

We consider a linear undamped vibrating system with one degree of freedom, excited by a sinusoidal force of constant amplitude. To this is attached a secondary system by means of a spring whose load-deflection characteristic is the sum of a linear and cubic term. It is desired to find optimum coefficients λ and ν for this coupling spring, such that one obtains as large a band of exciting frequencies as possible within which the vibration amplitude of the primary system is kept below unity. This choice is subject to the condition that the fundamental of the primary system response have zero amplitude at a preassigned expected excitation frequency Ω_0. The first approximation by the Duffing Iteration Method is used to obtain the response in terms of the system parameters. Optimum values of λ^2 and ν are expressed in terms of Ω_0^2. Using the optimum values corresponding to a particular value of Ω_0, the results of the synthesis are compared with those obtained by a more exact analysis and by an electronic differential analyzer. It is found that by the synthesis criterion used, the nonlinear absorber offers a significant advantage over the corresponding linear absorber.

NOMENCLATURE

$a_{1,2}$	response amplitudes for the linear case
$b_{1,2}$	approximate response amplitudes by Duffing's method
B	viscous damping coefficient for damping between main mass and inertial frame of reference
$c = B/2\sqrt{KM_1}$	damping ratio
$f(t)$	exciting force on main mass
f_m	maximum value of $f(t)$
K	spring constant for the linear spring between main mass and inertial frame of reference
$M_{1,2}$	masses of main mass and absorber
$S(1)$	suppression band width
$x_{1,2}$	displacements from equilibrium of main mass and absorber
$y_1 = x_1K/f_m$	dimensionless main mass displacement
$y_2 = (x_2 - x_1)K/f_m$	dimensionless relative displacement of absorber
λ^2	coefficient of the linear part of φ/μ
$\Lambda^2 = \mu\lambda^2$	
$\mu = M_2/M_1$	mass ratio
$M = 1 + 1/\mu$	
ν	coefficient of nonlinear part of φ
$\bar{\nu} = 3\nu/4\mu^3$	
$\bar{\nu}_F$	edge of forbidden region in a $\bar{\nu}\Omega^2$ phase plane

* Portions of a dissertation submitted to the Department of Applied Mechanics, Washington University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1] Mechanics Division, Naval Research Laboratory, Washington, D. C. Now at North American Aviation, Inc., Downey, Calif.

$\bar{\nu}_{U,L}$	upper and lower edges of the suppression band $S(1)$ in a $\bar{\nu}\Omega^2$ phase plane
$\tau = \omega t$	dimensionless time
$\varphi = \psi/\frac{1}{2}K$	
ψ	load-deflection function for the nonlinear spring
ω	circular frequency of the periodic function $f(t)$
$\omega_1 = \sqrt{K/M_1}$	natural frequency of main system
$\omega_2 = \omega_1\lambda_2$	natural frequency of linear absorber
$\Omega = \omega/\omega_1$	excitation frequency ratio
Ω_0	value of Ω which is to be expected in use
$\Omega_{L,U}$	frequencies corresponding to lower and upper edges of the suppression band $S(1)$
Ω_*	cross-over frequency

INTRODUCTION

The vibration characteristics of many machines and devices of practical interest are like those of a linear mass-spring system with one degree of freedom, excited by an external periodic force. It is well known that for certain frequencies of the exciting force, the response amplitude of such a system may become undesirably large.

The general problem of reducing vibrations has been under investigation for many years. Of the attempts in this direction, one of the most successful has been the linear dynamic vibration absorber, invented by Frahm early in the present century. In principle, a linear dynamic vibration absorber is an inertia member coupled to a vibrating system by suitable linear coupling members (usually a spring and viscous damper). It inhibits vibrations in the main system by providing a reaction force equal and opposite to the exciting force which causes the vibration.

However, it is not desirable to use a linear undamped dynamic vibration absorber in all cases. It is particularly unsuitable when the frequency of the exciting force is not fixed, but varies over a range of frequencies during normal operating conditions. The linear damped absorber attempts to overcome this limitation, but adds the practical difficulty of setting and maintaining a preassigned damping ratio, and acts (sometimes undesirably) to siphon off energy from the generator of the exciting force. A growing cognizance of these objections to the linear absorber is reflected in the nature of dynamic vibration absorbing devices patented during the last half-century, the more recent patents often having been ôn deliberately nonlinear devices. Such nonlinear devices apparently have been developed from the intuition and experimental efforts of their inventors. There seems to have been little systematic exploitation of nonlinearities, nor any attempt to use the known analytical tools of nonlinear mechanics on vibration absorber problems.

The problem of maintaining vibration absorber performance when the exciting frequency changes from its expected value, either intentionally or by accidental drift, can be serious under certain conditions. When the original system is not excited exactly at its natural frequency,

the resonance curve of the system after the addition of the absorber may be very steep. Then even a slight variation of excitation frequency could convert the vibration absorber to a vibration amplifier. One might try to correct the condition with a nonlinear absorber spring, which would effectively reduce the slope of the resonance curves in the neighborhood of the assigned working frequency, and would therefore permit a greater frequency drift without vibration amplification.

The rational synthesis of a nonlinear coupling element is the purpose of this paper. In particular, it is desired to synthesize a nonlinear spring characteristic which will permit satisfactory operation of the absorber over a wider range of frequencies than that permitted by a linear spring. For any device of engineering interest, synthesis is actually a two-phase operation. As applied to the undamped nonlinear vibration absorber, the first of these consists of a decision based on a mathematical treatment as to the kind of force-displacement characteristic the coupler should have. The second is a realization of this characteristic by a physical coupler, which is designed with many additional practical considerations in mind. The question of how to realize the spring characteristic dictated by the mathematical analysis will not be discussed, since the design phase does not lend itself to quantitative treatment. However, this does not mean that the realizability of the synthesized characteristic is merely a pious hope. Actually, a wide variety of spring characteristics are available, some of which are described in detail elsewhere (**2**, **3**, **4**, **5**, **7**, **8**).[2]

There is good reason for the *a priori* belief that nonlinear synthesis is mathematically feasible. Experimental work, particularly that of Ludeke (**6**), has indicated that some approximate methods of analysis lead, in the first approximation, to results in fair agreement with experiment. While his work concerns only one experimental arrangement, the same general result may hold for similar nonlinear physical systems. If so, a synthesis on the basis of first or second order approximations would lead to a design adequate for engineering practice.

Our purpose is to choose a nonlinear spring in such a way that the vibration of the main mass is made "small." Therefore, we must consider the *sense* in which it is to be small (that is, possible "criteria of synthesis"). This is by no means a trivial question, for the nature of the criterion used will be the determining factor in the results of the synthesis. The synthesis itself is confined to the choice of best parameters for cubic springs with no damping element between the main mass and the absorber, although the same methods are applicable to other forms. Using the first term approximation by the Duffing method, optimum design parameters are obtained as functions of a preassigned operating frequency. The response curve for the main mass as predicted by this first approximation is found to be in good agreement

[2] Boldface numbers in parentheses refer to the references appended to this paper.

with a more refined solution obtained by an iterative method, and with the results obtained on an electronic analog computer.

Although it is interesting to trace the historical development of linear and nonlinear vibration absorbers as physical devices, and of their mathematical descriptions, space limitations preclude its presentation here. The reader is referred to the unabridged dissertation for this historical background.

FORMULATION OF THE PROBLEM

It is supposed that the system consists of a vibrating mass and an attached dynamic vibration absorber, having the form shown in Fig. 1.

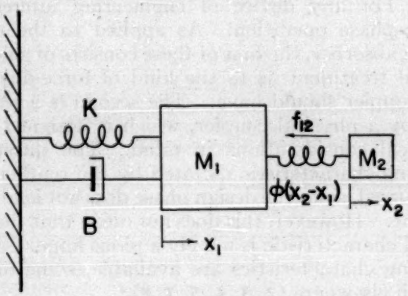

FIG. 1. The system.

The force-displacement characteristic of the (inertialess) nonlinear spring coupling the two systems is

$$f_{12} = \psi(x_2 - x_1), \tag{1}$$

where x_1 and x_2 are the respective displacements from equilibrium of the primary and secondary systems. The explicit assumptions which characterize the model system are:

A1. The main mass is coupled with one degree of freedom to an inertial reference frame by an inertialess linear spring and viscous damper.

A2. A sinusoidal exciting force is applied to the main mass.

A3. The frequency of the exciting force changes only over a limited range about some central frequency.

The system equations may be written by inspection as:

$$\begin{aligned} M_1\ddot{x}_1 + B\dot{x}_1 + Kx_1 + \psi(x_1 - x_2) &= f_m \sin \omega t \\ M_2\ddot{x}_2 + \psi(x_2 - x_1) &= 0. \end{aligned} \tag{2}$$

It is convenient to use a nondimensional form of Eq. 2. Introducing the variables defined in Nomenclature, and specializing the nonlinear

spring to the cubic form

$$\varphi(y_2) = \mu\lambda^2 y_2 + \nu y_2^3,$$

the equations of motion become

$$\begin{aligned} \Omega^2\ddot{y}_1 + 2c\Omega\dot{y}_1 + y_1 - \mu\lambda^2 y_2 - \nu y_2^3 &= \sin\tau \\ \mu\Omega^2(\ddot{y}_1 + \ddot{y}_2) + \mu\lambda^2 y_2 + \nu y_2^3 &= 0. \end{aligned} \tag{3}$$

The algebraic sign of ν is left open, positive values corresponding to a hard spring and negative values to a soft spring. The cubic absorber is perhaps the simplest generalization of the linear absorber, and is one which can be approximately realized by physical springs. In the sequel, we specialize to the undamped system, putting $c = 0$.[3] This is not a serious restriction, since the results for small damping are a simple perturbation on those for zero damping.

For reference, the solutions of Eq. 3 in the linear ($\nu = 0$) case are $a_1 = (\lambda^2 - \Omega^2)/\Delta$ and $a_2 = \Omega^2/\Delta$ where a_1 and a_2 are, respectively, the amplitudes of the main mass and relative sinusoidal motions, and $\Delta = (\lambda^2 - \Omega^2)(1 - \Omega^2) - \mu\lambda^2\Omega^2$. Certain characteristics of these solutions are of interest in motivating the investigation of nonlinear absorbers. The finite value of Ω at which $a_1 = 0$ is called the "cross-over point," and denoted by Ω_x. The slope of the main mass response curve at cross-over is not independent of λ, but is equal to $2/\mu\lambda^3$. In designing a linear absorber, one adjusts λ to make ω_2 equal to the expected excitation frequency Ω_0, over which the designer has no control. If it happens that this design value of λ is less than unity, as would be the case if the main system were originally excited somewhat below its resonant frequency,[4] the slope of the response curve at cross-over would be large. A small accidental variation of driving frequency could then shift the operating point to a position where vibrations would be amplified rather than suppressed by the absorber. The obvious cure, in case the driving force is not of constant frequency, is to make $\mu\lambda^3$ larger in designing the absorber; however, since the value of λ is specified by the tuning requirement, the only way to make this quantity larger is to make μ larger. Unfortunately, practical considerations usually limit the absorber mass to a small fraction of the main mass, so that this cannot be done. It must be concluded that under such conditions, a small variation in exciting frequency would generally be sufficient to render the linear absorber useless or even dangerous.

This fact, that the vibration absorber can act as a vibration amplifier under certain conditions as the excitation frequency changes from its design value, is a motivation for the investigation of nonlinear absorbers.

It is convenient to introduce the notion of a "suppression band." We put our attention on the neighborhood of the cross-over point, and

[3] In order to approach a steady state in the computer mechanization, some damping is required. For this reason, c was not set equal to zero at the outset.

[4] In this paper, we consider only this case $\Omega_0 \leq 1$.

restrict discussion to the frequency values between the resonance peaks of the response curves. There is an interval of Ω^2-values about Ω_x^2 for which $|a_1| \leqq 1$. We denote this interval by S, and call it the "suppression band for amplitude unity." In the sense that no excitation frequency below this band would give a main mass amplitude (without absorber) less than unity, the suppression band is a measure of the frequency range within which the addition of an absorber can be expected to improve the vibration condition of the main mass.

CRITERIA OF SYNTHESIS

Using Eq. 3, the parameters λ, μ, ν must be chosen to give the "best" nonlinear absorber. We suppose that μ is fixed by considerations of space or weight, leaving optimum values of λ and ν to be found. However, we must first choose a criterion for "best" performance—must say what is meant quantitatively by vibration reduction. There are several "natural" (that is, intuitively appealing) criteria, but because the choice is subjective, one cannot say that a certain criterion is "correct." We choose one by its appeal and ease of application, and as a final step, test the goodness of the optimum design to which the criterion leads.

Consider first that an expected operating frequency is known, as well as an expected range of frequency variation about this operating frequency. These quantities determine the location and width of a frequency band within which the main system response amplitude is to be kept as small as possible. One may take as a synthesis criterion for this case the condition that some function of amplitude be minimized over the given frequency range (for example, the maximum value, the mean magnitude, or the mean square amplitude). Of the examples mentioned, the last is more tractable analytically, but suffers the disadvantage that the criterion might permit relatively large response amplitudes over short intervals of the working band.

In the present paper, we consider a second problem, in which the expected working frequency is known and a maximum permitted response amplitude is known. (In particular, this may be $a_1 = 1$.) In this case, one may think of maximizing the suppression band about the operating frequency. A possible variation within this criterion is the placing of the working frequency within the suppression band. It is reasonable to make the cross-over point equal to the expected operating frequency, so the main mass is brought completely to rest under the expected excitation. However, other possibilities might be equally useful in certain problems.

The additional assumptions which limit the problem to be treated here are:

A4. There is no damping in the system.

A5. The nonlinear spring has a characteristic consisting of a linear plus cubic term.

A6. The synthesis criterion is the maximization of S, subject to the condition $\Omega_x = \Omega_0$.

A7. $\Omega_0 \leqq 1$.

Various generalizations of this basic problem are immediately evident. These are reserved for treatment elsewhere in order to avoid obscuring the essentially simple notions of nonlinear synthesis illustrated by the limited problem.

SYNTHESIS BY DUFFING'S METHOD

Equation 3 can be rewritten upon the introduction of

$$M = 1 + 1/\mu;\ \Lambda^2 = \mu\lambda^2 \tag{4}$$

in a form for iteration as

$$\begin{aligned} \Omega^2(\ddot{y}_1 + y_1) &= (\Omega^2 - 1)y_1 + \Lambda^2 y_2 + \nu y_2^3 + \sin\tau \\ \Omega^2(\ddot{y}_2 + y_2) &= (\Omega^2 - M\Lambda^2)y_2 + y_1 - \nu M y_2^3 - \sin\tau. \end{aligned} \tag{5}$$

It is to be understood that an approximate solution is to be used in the right hand side of Eq. 5, and that the next approximation is to be found by solving the resulting equations. Let us take as an approximation

$$y_1^{(0)} = b_1 \sin\tau;\ y_2^{(0)} = b_2 \sin\tau. \tag{6}$$

Then the differential equations for the next approximation are

$$\begin{aligned} \Omega^2[\ddot{y}_1^{(1)} + y_1^{(1)}] &= [(\Omega^2 - 1)b_1 + \Lambda^2 b_2 + 3\nu b_2^3/4 + 1]\sin\tau \\ &\quad - (\nu b_2^3/4)\sin 3\tau \\ \Omega^2[\ddot{y}_2^{(1)} + y_2^{(1)}] &= [b_1 + (\Omega^2 - M\Lambda^2)b_2 - 3\nu M b_2^3/4 - 1]\sin\tau \\ &\quad + (\nu M b_2^3/4)\sin 3\tau. \end{aligned} \tag{7}$$

The essential step is now to choose the coefficients of $\sin\tau$ equal to zero in order to avoid secular terms in the solution of Eq. 7 for $y_1^{(1)}$ and $y_2^{(1)}$. This gives the pair of algebraic equations

$$\begin{aligned} (\Omega^2 - 1)b_1 + \Lambda^2 b_2 + 3\nu b_2^3/4 + 1 &= 0 \\ b_1 + (\Omega^2 - M\Lambda^2)b_2 - 3\nu b_2^3 M/4 - 1 &= 0. \end{aligned} \tag{8}$$

Since these are functional relationships between the amplitudes b_1 and b_2 and the exciting frequency Ω, they give the nonlinear response curves for the system. It follows from Eq. 8 that

$$(\Omega_x^2 - \lambda^2)\Omega_x^4 - \bar{\nu} = 0, \tag{9}$$

where we have introduced $\bar{\nu}$ for the more cumbersome $3\nu/4\mu^3$.

It is easy to eliminate b_2 from Eq. 8. One finds that they reduce to the single equation

$$\bar{\nu} = \frac{\Omega^4\{\lambda^2 - \Omega^2 - b_1\Delta\}}{\{b_1[1 - (1+\mu)\Omega^2] - 1\}^3}. \tag{10}$$

If we now put $b_1 = 1$ and $b_1 = -1$, we have two relationships among λ^2, Ω^2, $\bar{\nu}$ which correspond to the upper and lower edges, respectively, of the suppression band. If we consider λ^2 to be held constant, for the moment, these relationships give curves in the $\bar{\nu}$-Ω^2 plane which describe the Ω^2-location of the edges of the suppression band as nonlinearity is increased. Denoting the upper and lower edges by appending subscripts U and L to $\bar{\nu}$,

$$\bar{\nu}_U = \frac{1}{(1+\mu)^3}[\Omega^2 - (1+\mu)\lambda^2]$$
$$\bar{\nu}_L = -\frac{\Omega^4[\lambda^2 - \Omega^2 + \Delta]}{[2-(1+\mu)\Omega^2]^3} \tag{11}$$

Consider Fig. 2, in which Eq. 11 is graphed in a $\bar{\nu}$-Ω^2 plane for the particular case $\lambda^2 = 0.75$ and $\mu = 0.1$. In it are shown the upper and

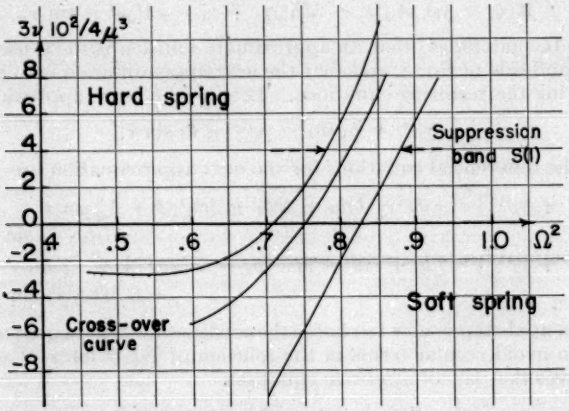

FIG. 2. Cross-over curve and edges of the $S(1)$ suppression band, for $\lambda^2 = 0.75$ and $\mu = 0.1$.

lower edges of the suppression band, as well as the cross-over curve given by Eq. 9. The suppression band can be seen to narrow as $\bar{\nu}$ is increased positively, at least initially. (A later widening is not important because the approximation method we have used becomes invalid for large values of $\bar{\nu}$.) On the other hand, it widens immediately as $\bar{\nu}$ is made negative. This means that our optimum spring will be one with a soft load-deflection characteristic.

The value of $\bar{\nu}$ which is optimum [5] would seem at first glance to be at the minimum of the $\bar{\nu}_L$-curve, since here the suppression band seems to reach all the way from the $\bar{\nu}_U$-curve down to $\Omega^2 = 0$. The situation is

[5] We denote optimum parameter values by superscript*.

actually much more complex, however. The choice of $\bar{\nu}^*$ is also subject to any physical limitations which may exist, but which may not be obvious in the representation of Fig. 2. The principal restriction we must place on the system is that the absorber vibration amplitude not be too large. Recall that a soft cubic spring has a load-deflection characteristic which becomes flat and then droops over as deflection is further increased. Because such a characteristic does not correspond to a physical spring (except of the Negator type), we must require that b_2 be kept smaller than the critical deflection at which the slope of the load-deflection curve vanishes. This amplitude is too large within certain regions of the $\bar{\nu}$-Ω^2 plane, on the soft side, and these are therefore forbidden regions for the system. Their boundary is found by setting $\varphi'(b_2) = 0$ which corresponds to $b_2^2 = \lambda^2/4\mu^2|\bar{\nu}|$ or

$$\bar{\nu}_F = -\frac{\lambda^2}{4\mu^2\Omega^4}\{\Delta - \lambda^2[1 - (1+\mu)\Omega^2]\}^2. \tag{12}$$

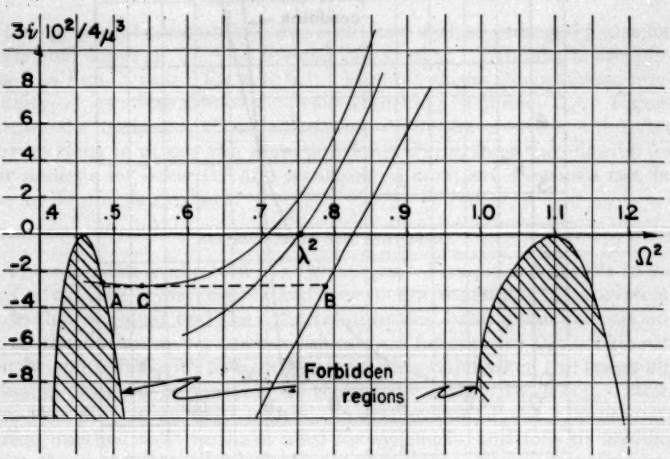

FIG. 3. Forbidden regions in relation to the $S(1)$ suppression band, for $\lambda^2 = 0.75$ and $\mu = 0.1$.

In Fig. 3, the forbidden regions are put into the same phase diagram as Fig. 2, namely that for $\lambda^2 = 0.75$, $\mu = 0.1$. From this figure it appears that the widest $S(1)$ suppression band is obtained at the $\bar{\nu}$-value for which the $\bar{\nu}_L$ *vs.* Ω^2 curve enters the forbidden regions. Now, however, we see a second limitation. If the $\bar{\nu}_L$-curve has a minimum at point C to the right of the forbidden region, then the optimum $\bar{\nu}$-value is that which corresponds to the minimum point. In this case, the optimum suppression band would reach from point A to B. (If a $\bar{\nu}$-value above

the level of C were to be used, part of the line AB would lie in a region of amplitude less than -1; or, in other words, the band would be split into two disconnected parts.) The only question, then, in deciding the optimum value of ν, is whether the minimum of the $\bar{\nu}_L$-curve occurs within or to the right of the forbidden region.

The question can be decided on the basis of a rather tedious argument, the details of which are not important for the present purposes.

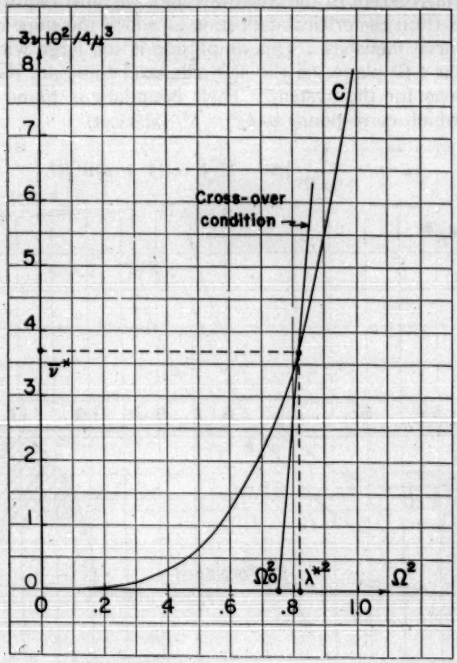

FIG. 4. Graphical determination of the optimum parameter values $\bar{\nu}^*$ and λ^{*2} as functions of Ω_0^2.

It suffices to say that under rather general conditions, the forbidden region lies to the left of the minimum. Therefore, we can take the $\bar{\nu}$-value corresponding to point C (of Fig. 3) as $\bar{\nu}^*$. Even if a more rational choice were to be on the basis of the intersection of the forbidden region with the suppression band edge, it is unlikely that the band width would be sensitive to small changes in optimum values. In any event, the errors inherent in the approximation method would be likely to overshadow such small differences.

The complete synthesis now rests on the two propositions that: (1) a working frequency $\Omega_x = \Omega_0$ is preassigned; and (2) the optimum value of $\bar{\nu}$ is that corresponding to a minimum of the $\bar{\nu}_L$-curve. This minimum occurs at a value of Ω^2 which satisfies

$$(1 + \mu)\Omega^6 - 8\Omega^4 + 4[3 + (1 + \mu)\lambda^2]\Omega^2 - 8\lambda^2 = 0. \qquad (13)$$

Equations 11 and 13 taken together give a parametric relationship between the optima $\bar{\nu}^*$ and λ^{*2}, with Ω^2 as the parameter. The second relationship between these optima is that derived from Eq. 9,

$$\bar{\nu}^* = (\Omega_0^2 - \lambda^{*2})\Omega_0^4. \qquad (14)$$

Let us suppose that the parametric relationship mentioned is laid off graphically, for constant μ giving a curve C in a $\bar{\nu}^* - \lambda^{*2}$ plane as shown in Fig. 4. Now, given Ω_0, a straight line can be laid off in this plane according to Eq. 14, as shown in that figure for $\Omega_0^2 = 0.75$. The intersection of the straight line with curve C gives the optimum values $\bar{\nu}^*$ and λ^{*2}.

This graphical computation has been done with an enlarged figure for a restricted range of Ω_0^2, and several values of μ. The results are presented as Figs. 5 and 6 for $\mu = 0.1$. Figure 5 shows the optimum nonlinearity, ν^*, as a function of the desired working frequency Ω_0^2. Figure 6 shows the optimum λ^{*2} as a function of Ω_0^2 for $\mu = 0.1$, the other μ-curves lying in nearly the same position. From these two figures (or their analogs for other Ω_0^2 and μ ranges), a complete synthesis can be done by Duffing's method.

EVALUATION OF THE SYNTHESIS

The synthesis evaluation has two stages. First, it is desirable to see what advantage, if any, is obtained over the corresponding linear system. To do this, we must find the actual suppression band width corresponding to the optimum parameter values as a function of Ω_0^2. This can then be compared with the linear suppression band when the linear absorber is designed to function at the same exciting frequency. However, this step is entirely within the framework of the approximate Duffing method that we have used for synthesis, and tells us nothing about the real value of the nonlinear absorber unless we can first say that the approximate method gives a response curve close to that obtained by a more exact method. Thus, a second stage in testing the goodness of the synthesis might consist in choosing an operating frequency Ω_0 at which to evaluate the synthesis (for clearly it is impractical to evaluate the synthesis for all frequencies), and comparing its results with those obtained by a more exact numerical process and by an electronic differential analyzer.

Consider first the comparison between the predicted nonlinear results and those for a linear absorber. The suppression band for the nonlinear absorber can be found in the following way. For each expected working

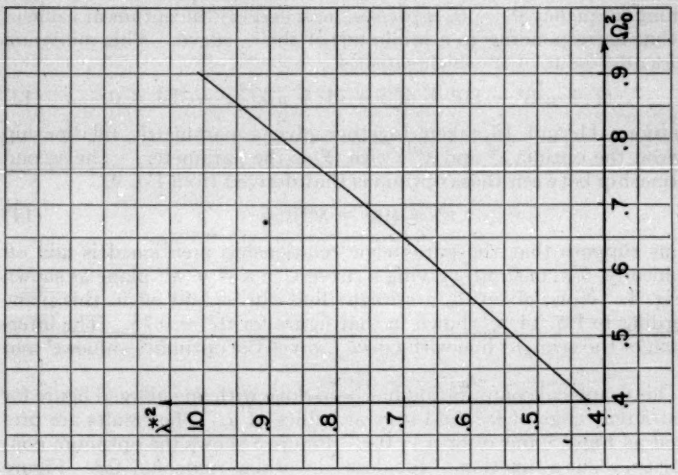

FIG. 6. λ^{*2} as a function of Ω_0^2.

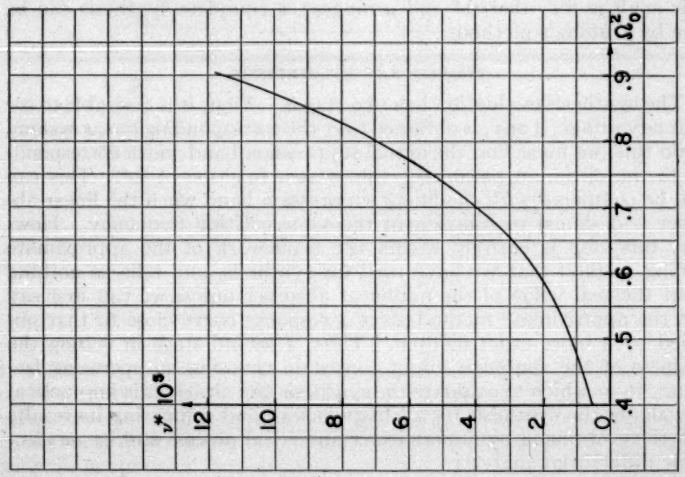

FIG. 5. ν^* as a function of Ω_0^2.

frequency Ω_0, the corresponding optimum ν^* or λ^{*2} is shown by Fig. 5 or 6. Returning with either of these values to the curve C of Fig. 4 (which was obtained from parametric equations in the Ω_L^2 corresponding to the lower edge of the suppression band), it is not difficult to find Ω_L^2 as a function of Ω_0^2. The frequency Ω_U^2 corresponding to the upper edge is found directly from Eq. 11a, in which one uses the optimum parameter values. Thus, one obtains a complete picture of the band as a function of Ω_0^2 in the nonlinear case. In the corresponding linear case, it can be shown that

$$2\Omega_L^2 = 2 + \mu M\lambda^2 - \sqrt{(2 + \mu M\lambda^2)^2 - 8\lambda^2} \qquad (15)$$

$$\Omega_U^2 = \mu M\lambda^2.$$

Here, of course, it is to be understood that $\lambda = \Omega_0$, since it is under this condition that the linear absorber be designed to operate at the same exciting frequency as the nonlinear absorber.

Figure 7 shows the suppression band for the linear and nonlinear

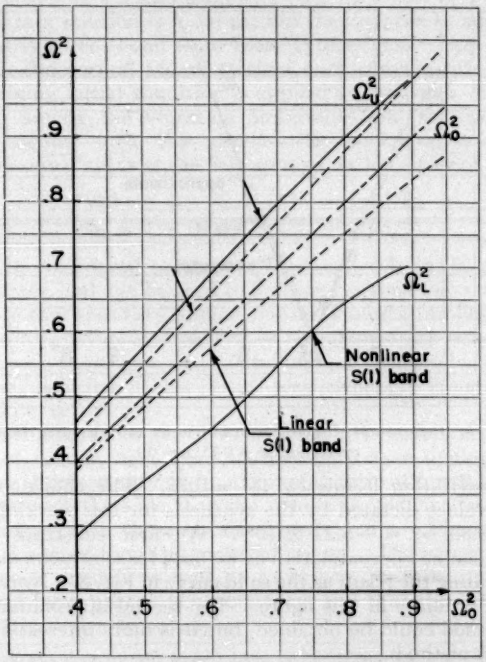

FIG. 7. Comparison of the suppression bands for the optimum nonlinear absorber and the corresponding linear absorber.

cases, using optimum parameter values, as a function of Ω_0^2. The width of these bands, as well as their relation to the cross-over frequency, can be seen from this figure. Only a restricted range of Ω_0^2 is considered, since we have previously limited discussion to $\Omega_x^2 \leq 1$, and since for small Ω_0^2 the suppression bands narrow and become indistinguishable. The nonlinear suppression band is seen to be much wider than the corresponding linear band, and entirely contains the latter (except for a small portion near the upper edge for close to unity). A particular improvement is the departure of the lower edge from the cross-over frequency, which is precisely the advantage sought over the linear absorber.

We can obtain some assurance that the synthesis as presented is satisfactory if we examine a special case in the light of other methods of treating the nonlinear equations. Let us consider the response for the

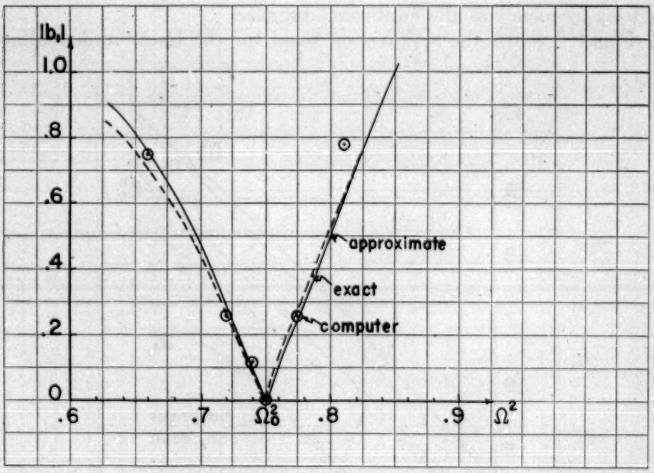

FIG. 8. Comparison of approximate with exact and computer response curves for $\Omega_0^2 = 0.75$, $\mu = 0.1$.

particular excitation frequency $\Omega_0^2 = 0.75$. The choice of optimum parameter values dictated by the synthesis for this working frequency are $\lambda^{*2} = 0.82$, $\nu^* = -5.27 \times 10^{-5}$. We first construct a response curve by the same approximation as we used for synthesis, using Eqs. 13 and representing the result as the solid curve in Fig. 8. Note that $|b_1|$ is used as the ordinate in this figure. The second approximation by the Duffing method could be obtained, but it is more interesting to use an independent method.

We extend to systems with two degrees of freedom the modified

Picard iteration method which has recently been used by Brock (1). We advance the time scale by $\pi/2$ in order to have the same form for the iteration as (1), and write the equations of motion as

$$\begin{aligned} \Omega^2\ddot{y}_1 &= \cos\tau + (\Lambda^2 y_2 + \nu y_2^3) - y_1 \\ -\Omega^2\ddot{y}_2 &= \cos\tau + M(\Lambda^2 y_2 + \nu y_2^3) - y_1. \end{aligned} \tag{16}$$

It is understood that optimum values of λ^2 and ν are to be used here. As associated conditions, we assume that $\dot{y}_1(o) = \dot{y}_2(o) = 0$, $y_1(\pi/2) = y_2(\pi/2) = 0$, and $y_1(o) = b_1$, $y_2(o) = b_2$. The method consists of a formal integration of Eqs. 16 over the range $(0, \tau)$, followed by another over $(\tau, \pi/2)$. The result is

$$\begin{aligned} -\Omega^2 y_1 &= \int_\tau^{\pi/2} dv \int_0^v [\cos w + (\Lambda^2 y_2 + \nu y_2^3) - y_1] dw \\ \Omega^2 y_2 &= \int_\tau^{\pi/2} dv \int_0^v [\cos w + M(\Lambda^2 y_2 + \nu y_2^3) - y_1] dw. \end{aligned} \tag{17}$$

An approximate solution is to be used in the integrands, and a new approximation is to be found from these expressions. This can be continued as in the normal Picard method until convergence is obtained. If the remaining initial condition is applied at each step, one obtains a relationship among the vibration amplitudes and the excitation frequency. Starting with the approximate solutions $y_1 = b_1 \cos\tau$, $y_2 = b_2 \cos\tau$ one obtains at the first approximation a pair of equations

$$\begin{aligned} -\Omega^2 b_1 &= 1 - b_1 + (\Lambda^2 b_2 + 7\nu b_2^3/9) \\ \Omega^2 b_2 &= 1 - b_1 + M(\Lambda^2 b_2 + 7\nu b_2^3/9). \end{aligned} \tag{18}$$

These can be considered parametric equations relating b_1 and Ω^2, with b_2 as parameter, and can be used to find the response curve for the main mass. Note that these are similar, but not identical, to Eqs. 8. It can be shown that another iteration does not change the result to within graphing errors, so that we may consider Eq. 8 as a good approximation to the "true" solution. This "true" response curve is shown in Fig. 8 as a dotted curve.

As a final test of the goodness of the solutions obtained above, the equations of motion have been mechanized on an analog computer.[6] In order to attain a steady state, a damping ratio of $c = 0.1$ was used in obtaining the solutions, so that the results should not be expected to coincide exactly with those obtained analytically. The responses obtained are shown in Fig. 8 as encircled points. The agreement of these results is good enough to lead us to believe that the approximation method used for synthesis is satisfactory, and that the values given in Figs. 6 and 7 are nearly optimum.

[6] This work was done by Mr. Robert Light on the Naval Research Laboratory REAC computer.

A closing remark is in order regarding the return to the original variables. The optimum values of ν^* are so small that the nonlinear term might appear negligible compared to the linear term. Remember, however, that the load-deflection curve for the nonlinear spring is, in terms of the original variables,

$$f_{12} = \mu K(x_2 - x_1)[\lambda^{*2} + \nu^* K^2(x_2 - x_1)^2/\mu f_m{}^2].$$

Since, in general, $K \gg f_m$, the nonlinear term may well be of the order of the linear term.

REFERENCES

(1) J. E. BROCK, "An Iterative Numerical Method for Nonlinear Vibrations," *J. Appl. Mech.*, Vol. 18, pp. 1–11 (1951).

(2) S. P. CLURMAN, "The Design of Nonlinear Leaf Springs," *Trans. ASME*, Vol. 73, pp. 155–161 (1951).

(3) R. J. COLLINS, U. S. Patent No. 1,037,729, Sept. 3, 1912.

(4) W. J. COOK AND P. C. CLARK, "The Negative Spring—A Basic New Elastic Member," *Prod. Engr.*, July, 1949, pp. 136–140.

(5) W. E. JOHNSON, "The Load-Deflection Characteristics of Initially Curved Flexural Springs," *J. Appl. Mech.*, Vol. 4, pp. A119–A127 (1937).

(6) C. A. LUDEKE, "Experimental Investigation of Forced Vibration in a Mechanical System Having a Nonlinear Restoring Force," *J. Appl. Phys.*, Vol. 17, pp. 603–609 (1946).

(7) A. M. WAHL, "Mechanical Springs," Cleveland, Penton Publ. Co., 1944.

(8) F. J. WINKLER, U. S. Patent No. 1,096,582, May 12, 1914.

PROPERTIES OF AN EXPONENTIAL SIZE-FREQUENCY DISTRIBUTION OF FINITE EXTENT

BY

LUDWIK SILBERSTEIN [1]

The size-frequency distribution among the grains of some emulsions may be represented, schematically, by a simple exponential function, with sufficient accuracy as to the sensitometric properties of the emulsion. With such an object in view we have frequently applied it in the past, especially in connection with the quantum theory of exposure.

In doing so, however, we have always assumed (for the sake of computational simplicity) that the sizes a (projected areas) of all grains presented extend from 0 to ∞. If N be the total number of grains and $Nf(a)\,da$ the number of grains of size a to $a + da$, the exponential distribution function, which manifestly must satisfy the requirement

$$\int_0^\infty f(a)\,da = 1,$$

assumes then the simple form

$$f(a) = \frac{1}{\bar{a}}\,e^{-a/\bar{a}},$$

where $\bar{a} = \int_0^\infty af(a)\,da$ is the mean grain size.

Actually, the grains whose size is several times the mean size are but sporadic. In many emulsions there is scarcely one grain in a thousand having a size $a = 5\bar{a}$ or even $4\bar{a}$.

It has, therefore, seemed interesting and, perhaps useful for future applications, to construct the exponential distribution function for the case in which a_m, the largest size present at all bears a certain, given finite ratio to the mean size, say

$$a_m = m\bar{a}, \tag{1}$$

where m is some positive number, greater than unity, of course.

Our simple problem may be formulated as follows: Given the mean size $\bar{a}$ and the ratio $a_m : \bar{a} = m$, find all the parameters of the exponential distribution function.

The most general form of such a function is

$$f(a) = Ce^{-\kappa a}. \tag{2}$$

The problem consists, then, in determining the parameters C and κ through $\bar{a}$ and a_m or $\bar{a}$ and m.

[1] Deceased; formerly, Research Consultant, Rochester, N. Y.

Now, by the very meaning of $f(a)$, we must have

$$\frac{1}{C} = \int_0^{\bar{a}_m} e^{-\kappa a} da = \frac{1}{\kappa}\int_0^{\kappa\bar{a}_m} e^{-x}\, dx,$$

that is,

$$\frac{1}{C} = \frac{1}{\kappa}(1 - e^{-\kappa a_m})$$

or, by (1),

$$C = \frac{\kappa}{1 - e^{-m\kappa\bar{a}}}. \tag{3}$$

Next, the mean grain size is

$$\bar{a} = C\int_0^{m\bar{a}} ae^{-\kappa a}\, da = \frac{C}{\kappa^2}\int_0^{m\kappa\bar{a}} xe^{-x}dx$$

or

$$\bar{a} = \frac{C}{\kappa^2}[1 - (1 + m\kappa\bar{a})e^{-m\kappa\bar{a}}].$$

Thus, and by (3),

$$\kappa\bar{a} = 1 - \frac{m\kappa\bar{a}}{e^{m\kappa\bar{a}} - 1}. \tag{4}$$

If $m \to \infty$ (as in our previous investigations), then (3) reduces at once to $C = \kappa$ and (4) becomes $\kappa\bar{a} = 1$, so that

$$f(a) = \frac{1}{\bar{a}}\, e^{-a/\bar{a}},$$

as stated at the outset.

For any finite value of the positive number $m = a_m : \bar{a}$, (4) is a transcendental equation for $\kappa\bar{a}$. Let us try to solve it, approximately.

Putting

$$x = e^{m\kappa\bar{a}},$$

we have

$$F(x) \equiv \frac{x + (m - 1)}{x - 1}\log x = m, \tag{5}$$

a transcendental equation for x. We may solve it by successive approximations. Starting with a reasonable $x = x_1 < e^m$ as our first approximation, we shall have, for the second approximation x_2,

$$\log x_2 = m\frac{x_1 - 1}{x_1 + m - 1};$$

similarly,

$$\log x_3 = m\frac{x_2 - 1}{x_2 + m - 1},$$

and so on, for example, for $m = 4$, when $\log x = \dfrac{4(x - 1)}{x + 3}$, start with

$x_1 = 30$. Then the second and the following approximations will be found to be

$$33.7, \qquad 35.3, \qquad 36.0, \qquad 36.3, \qquad 36.4,$$

converging rapidly enough to the correct value which, to three significant figures is

$$x = 36.5.$$

In fact, by (5), $F(36.5) = 4.007$, showing but a negligible excess over $m = 4$.

From $x = e^{4\kappa\bar{a}} = 36.5$ follows $\kappa\bar{a} = 0.899$ and therefore,

$$C = \frac{\kappa}{1 - \frac{1}{x}} = 0.924/\bar{a},$$

so that, ultimately

$$f(a) = \frac{0.924}{\bar{a}} e^{-0.899a/\bar{a}}. \qquad (m = 4)$$

In much the same way we find, for $m = 3$

$$x = 8.55, \text{ whence } \kappa\bar{a} = 0.715$$

and, the complete frequency function,

$$f(a) = \frac{0.810}{\bar{a}} e^{-0.715a/\bar{a}}. \qquad (m = 3)$$

With increasing $m = a_m$: $\bar{a}$, the coefficients C and κ tend, rapidly enough, to their asymptotic value $1/\bar{a}$.

These successive approximations, however, are somewhat laborious. A much more expedient way is to insert Eq. 5 writing it

$$m = \frac{(x - 1) \log x}{x - 1 - \log x} \qquad (6)$$

and computing m for a set of freely chosen x-values, most conveniently, for simple powers of e. Thus, for example, for $x = e$ we have

$$m = \frac{e - 1}{e - 2} = 2.392, \text{ whence } \kappa\bar{a} = \frac{1}{m} = 0.418.$$

Similarly, for $x = e^2$

$$m = \frac{2(e^2 - 1)}{e^2 - 3} = 2.911, \therefore \kappa\bar{a} = \frac{2}{m} = 0.687;$$

for $x = e^3$,

$$m = \frac{3(e^3 - 1)}{e^3 - 4} = 3.560, \therefore \kappa\bar{a} = \frac{3}{m} = 0.843;$$

for $x = e^4$,

$$m = \frac{4(e^4 - 1)}{e^4 - 5} = 4.32, \therefore \kappa\bar{a} = \frac{4}{m} = 0.926,$$

and so on.

Having, in this manner, computed a number of "points" $(m, \kappa\bar{a})$, a smooth curve has been drawn through them in Fig. 1. From this curve a sufficiently accurate value of $\kappa\bar{a}$ may be read off for any desired value of $m = a_m : \bar{a}$.

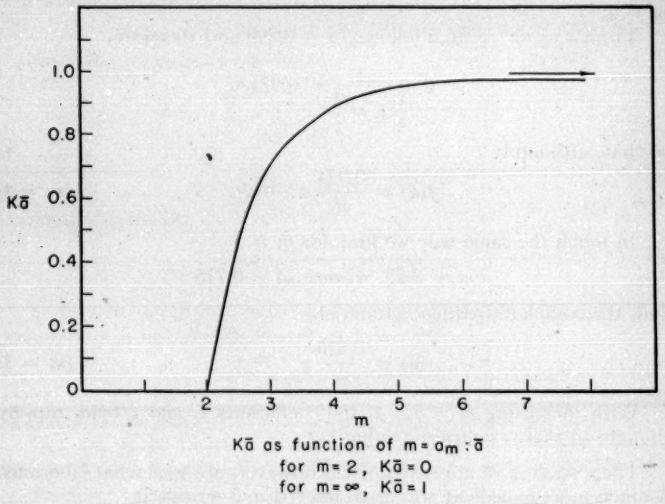

FIG. 1.

Notice that for $x = e^{1/n}$, where $n \to \infty$ we have, by (6)

$$m = \frac{e^{1/n} - 1}{n(e^{1/n} - 1) - 1} = \frac{1 + \frac{1}{2n} + \frac{1}{6n^2} + \cdots}{n(1 + \frac{1}{2n} + \frac{1}{6n^2} + \cdots - 1)} \to \frac{2\left(1 + \frac{1}{2n}\right)}{1 + \frac{1}{3n}}$$

or

$$m = 2\left(1 + \frac{1}{6n}\right) \to 2.$$

At the same time, $\kappa\bar{a} = \frac{1}{m}\log x = \frac{1}{mn} \to 0$. In fine, when m tends to 2, $\kappa\bar{a}$ tends to zero (that is, $\kappa \to 0$), and $f = C = \frac{1}{2\bar{a}}$. On the other extreme, for $m \to \infty$, $\kappa\bar{a}$ tends to 1.

In fine, our curve representing the dependence of $\kappa\bar{a}$ on m starts from 0 at $m = 2$ and mounts asymptotically (for $m \to \infty$) to $\kappa\bar{a} = 1$.

We have found that $m = 2$ leads to $\kappa\bar{a} = 0$, that is, to $\kappa = 0$ and thus to

$$f(a) = C = \text{constant}.$$

In words: An exponential frequency function extending just from $a = 0$ to $a_m = 2\bar{a}$ *reduces to a constant*, viz.

$$f = C = \frac{1}{2\bar{a}}.$$

Since this result seems rather unexpected and puzzling, it may be well to show here explicitly that *the only non-negative solution* of Eq. 4 in the case $m = 2$, that is, of the equation

$$e^{2\kappa\bar{a}} - 1 = \frac{2\kappa\bar{a}}{1 - \kappa\bar{a}} \tag{4.2}$$

is $\kappa\bar{a} = 0$.

With $\kappa\bar{a} = 2$; then the equation is

$$e^{2z} - 1 = \frac{2z}{1 - z}$$

or

$$2z + \frac{4z^2}{2!} + \frac{8z^3}{3!} + \cdots = \frac{2z}{1 - z}. \tag{A}$$

Now, one solution of Eq. A is $z = 0$. If there is some other solution $z \neq 0$, it must satisfy the equation

$$1 + z + \tfrac{2}{3}z^2 + \tfrac{1}{3}z^3 + \cdots = \frac{1}{1 - z} = 1 + z + z^2 + z^3 + \cdots,$$

whence

$$\tfrac{2}{3}z^2 + \tfrac{1}{3}z^3 + \cdots = z^2 + z^3 + z^4 + \cdots,$$

or

$$\tfrac{2}{3} + \tfrac{1}{3}z + \cdots = 1 + z + z^2 + \cdots,$$

or, ultimately,

$$\tfrac{1}{3} + z\left(1 - \frac{8}{24}\right) + z^2\left(1 - \frac{16}{5!}\right) + z^3\left(1 - \frac{3^2}{6!}\right) + \cdots = 0,$$

that is,

$$\tfrac{1}{3} + \tfrac{2}{3}z + \frac{13}{15}z^2 + \frac{43}{45}z^3 + \cdots = 0.$$

The coefficients of 2^4, 2^5, etc., are all *positive* (tending to $+1$). Thus the equation *cannot* be satisfied by any positive z. In other words, *the only non-negative* solution of (4.2) is $z = 0$ or $\kappa = 0$, which was to be proved. Such is also the case for $m < 2$. The proof may be left to the reader. Manifestly, $m > 1$, by the meaning of m.

Louvred Windows.—Few things have remained status quo during the past 450 years, but the "double hung" window, most commonly used today, has remained essentially unchanged since it made its first appearance in England somewhere in the 1400's.

Here in America, this window design based on an upper and lower sash that slides up and down has been used for centuries, despite its many shortcomings. Besides admitting light and air, the window has the obvious function of creating ventilation. When considering the important air flow factor, the "double hung" window leaves much to be desired. Offering only a 50 per cent maximum opening, this window permits the air to enter and continue only on a horizontal level inside the room, or in the same direction as the outside source. By adjustments of the sash, the air can enter the room at various horizontal levels from top to bottom.

Only recently, since the Sun Sash Company (N. Y. C.) introduced their louvred window in this country, has a completely new window design become available to architects and builders. Blending with the current architectural trend toward functional design, the louvre principle affords 100% ventilation when fully opened. Even when partially opened, drafts are eliminated because the air is directed inside to any upward angle at which the glass louvres are adjusted. When partially open, the cool outside air is directed toward the ceiling, displacing collected warm air and maintaining more uniform room temperatures. In addition, the Sun-Sash window possesses the unique ability of directing the air horizontally or downwards as well, allowing installation at different heights with little or no effect upon the maximum benefits to be received from it. Many hospitals have realized this important advantage by choosing the louvred window.

An examination of the Sun-Sash window indicates that it consists of two heavy galvanized steel uprights with rotatable pockets mounted on bronze bearings. These pockets are designed to accommodate 6″ glass louvred blades. A single lever, operating through a concealed linkage, permits all the louvres to be opened simultaneously by applying finger-tip pressure. These louvres remain in any desired position and lock automatically in the close position. A very broad comparison can be made between the operating theories of the louvred window and the venetian blind.

Strong support for the Sun-Sash window comes from the lady of the house on four important points. First, children cannot fall out and secondly, burglars cannot enter. The third point—left open during rain storms, the Sun Sash window practically eliminates the danger of water infiltration and consequent damage to draperies and furniture. Even in the closed position, condensation or "sweating" does not occur on the inside surfaces. Another factor she appreciates is the ease with which outside surfaces are cleaned from the inside.

The low installation price of the louvred window has been a very strong contributing factor to its current acceptance by the building industry. Also, broken window panes are easily replaced without special tools or skills. It's only necessary to slide a new pane into the pocket.

SEPARABILITY IN A CLASS OF COORDINATE SYSTEMS

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

SUMMARY

A study is made of separability conditions for the Helmholtz and Laplace equations. Attention is confined to the most useful case for physical applications; namely,

(*a*) The coordinate system is either cylindrical or it has rotational symmetry, and

(*b*) The potential is independent of the third space variable.

Necessary and sufficient conditions are tabulated, by which one can easily determine if a proposed coordinate system will allow solutions by the method of separation of variables.

INTRODUCTION

Separation of variables constitutes a powerful tool for the solution of the partial differential equations of mathematical physics (1).[3] Unfortunately, however, many coordinate systems do not allow separation. For instance, Eisenhart (2) has shown that, in euclidean 3-space with potential a function of all three space variables, simple separability of the Schrödinger equation is possible in only eleven coordinate systems. And of the eleven, three (the general conical, paraboloidal, and ellipsoidal) are so complicated that they have little practical utility.

The apparatus of physics and engineering customarily exhibits some kind of symmetry; so in dealing with the field problems that arise in these subjects, one will naturally employ either some form of cylindrical coordinate system or an axially symmetric system. Also, in many cases the potential is a function of only two variables instead of three. It seems reasonable that these simplified conditions should lighten the separability restrictions that apply to the general three-dimensional problem (3). The present paper shows how separability conditions for the general case are modified when the potential becomes a function of only two variables.

We shall confine ourselves to euclidean 3-space and to orthogonal coordinate systems. It is well known that separation is never possible unless the coordinate surfaces constitute a triply orthogonal set, so all skew systems can be discarded. Two types of coordinate systems will be considered:

(*a*) *Cylindrical Systems.* The coordinate surfaces consist of a family of parallel planes and two families of cylinders, the generators of

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Department of Mathematics, University of Connecticut, Storrs, Conn.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

which are orthogonal to the planes. A cylindrical system can be generated by taking any orthogonal map in a plane and translating the plane in a direction perpendicular to itself.

(*b*) *Rotational Systems*. Rotational or axially symmetric coordinate systems may be generated by rotating an orthogonal map about an axis. The axis of rotation is usually an axis of symmetry for the plane map. The coordinate surfaces consist of two axially symmetric families which intersect orthogonally, plus a set of half-planes which terminate on the axis of rotation.

We shall impose the further restriction that the potential is a function of only two variables. In cylindrical coordinates (u^1, u^2, z), the generator of the cylinders will be taken parallel to the z-axis and the potential will be independent of z. In rotational coordinates (u^1, u^2, ψ), the z-axis is taken as the axis of rotation and ψ is the angle about this axis. With either type of coordinate system, the potential may be written

$$\varphi = \varphi(u^1, u^2).$$

It is convenient to distinguish two classes of separability: *simple separability and R-separability* (3). A partial differential equation in n independent variables is simply separable if, when the potential is expressed as the product

$$\varphi = U^1(u^1) \cdot U^2(u^2) \cdots U^n(u^n),$$

the equation allows separation into n ordinary differential equations. R-separability occurs when the assumption

$$\varphi = \frac{U^1(u^1) \cdot U^2(u^2) \cdots U^n(u^n)}{R(u^1, u^2, \cdots u^n)}$$

($R \neq$ const.) leads to the separation of the equation into n ordinary differential equations. Note that, according to these definitions, an equation that allows separation with $R =$ const. is simply separable and is *not* R-separable.

Separability conditions will be investigated for the following cases:

1. R-separability of the Helmholtz equation
   - (*a*) Cylindrical coordinates
   - (*b*) Rotational coordinates
2. R-separability of the Laplace equation
   - (*a*) Cylindrical coordinates
   - (*b*) Rotational coordinates
3. Simple separability of the Helmholtz equation
   - (*a*) Cylindrical coordinates
   - (*b*) Rotational coordinates
4. Simple separability of the Laplace equation
   - (*a*) Cylindrical coordinates
   - (*b*) Rotational coordinates

As stated previously, the separability conditions determined in this paper apply only to the special case where the potential is independent of the third space variable.

1. THE COORDINATE SYSTEMS

Triply orthogonal sets of surfaces can be obtained in various ways. For instance, the investigator may study the properties of confocal quadrics or he may investigate cyclides, as was done by Bôcher (4). One of the most convenient ways of finding new coordinate systems, however, is by complex-plane transformations, employing either arbitrary functional transformations or using the Schwarz-Christoffel method. The resulting map is then translated or rotated to give a triply orthogonal family of coordinate surfaces.

Let

$$z = \mathfrak{F}(w), \tag{1}$$

where $z = x + iy$, $w = u + iv$, and where $\mathfrak{F}$ is any analytic function.[4] The real and imaginary parts are separated, giving two equations

$$\begin{cases} x = \xi_1(u, v), \\ y = \xi_2(u, v). \end{cases} \tag{2}$$

These equations represent (in the z-plane) a family of curves $u = \text{const.}$, which intersect orthogonally the family $v = \text{const.}$

We now form coordinate systems for each function $\mathfrak{F}$: a *cylindrical* system, and one or two *rotational* systems. The cylindrical system (u^1, u^2, u^3) is represented by the equations,

$$\begin{cases} x = \xi_1(u^1, u^2), \\ y = \xi_2(u^1, u^2), \\ z = u^3. \end{cases} \tag{3}$$

The z-axis is always parallel to the generators of the cylinders, and the coordinate surfaces are $u^1 = \text{const.}$, $u^2 = \text{const.}$, $u^3 = \text{const.}$

The rotational system (u^1, u^2, ψ) is

$$\begin{cases} x = \xi_1(u^1, u^2) \cdot \cos\psi, \\ y = \xi_1(u^1, u^2) \cdot \sin\psi, \\ z = \xi_2(u^1, u^2) \end{cases} \tag{4}$$

or

$$\begin{cases} x = \xi_2(u^1, u^2) \cdot \cos\psi, \\ y = \xi_2(u^1, u^2) \cdot \sin\psi, \\ z = \xi_1(u^1, u^2). \end{cases} \tag{5}$$

The former represents rotation about what was originally the y-axis; the latter, about what was originally the x-axis. The z-azis of the rotational system is always taken as the axis of rotation. Rotation is possible also about other lines in the z-plane.

[4] Equation 1 imposes no restriction on our coordinate systems. $\mathfrak{F}$ need not be confined to elementary functions but may include functions defined by series and by integrals.

Since $\mathfrak{F}$ is an analytic function, the Cauchy-Riemann equations apply:

$$\frac{\partial \xi_1}{\partial u^1} = \frac{\partial \xi_2}{\partial u^2}, \qquad \frac{\partial \xi_1}{\partial u^2} = -\frac{\partial \xi_2}{\partial u^1}. \tag{6}$$

From these equations are obtained the relations (which will be used later):

$$\frac{\partial^2 \xi_i}{(\partial u^1)^2} + \frac{\partial^2 \xi_i}{(\partial u^2)^2} = 0 \qquad (i = 1, 2) \tag{7}$$

and

$$\left(\frac{\partial \xi_1}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^1}\right)^2 = \left(\frac{\partial \xi_1}{\partial u^2}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^2}\right)^2$$
$$= \left(\frac{\partial \xi_1}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_1}{\partial u^2}\right)^2 = \left(\frac{\partial \xi_2}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^2}\right)^2. \tag{8}$$

The metric coefficients are evaluated by means of the familiar equation,

$$g_{ii} = \left(\frac{\partial x}{\partial u^i}\right)^2 + \left(\frac{\partial y}{\partial u^i}\right)^2 + \left(\frac{\partial z}{\partial u^i}\right)^2 \qquad (i = 1, 2, 3) \tag{9}$$

or from

$$g^{ii} = \left(\frac{\partial u^i}{\partial x}\right)^2 + \left(\frac{\partial u^i}{\partial y}\right)^2 + \left(\frac{\partial u^i}{\partial z}\right)^2 \tag{10}$$

and

$$g_{ii} = \frac{1}{g^{ii}}. \tag{11}$$

For the cylindrical system, Eq. 3, the metric coefficients of Eq. 9 assume the simple form,

$$g_{11} = \left(\frac{\partial \xi_1}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^1}\right)^2,$$
$$g_{22} = \left(\frac{\partial \xi_1}{\partial u^2}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^2}\right)^2, \qquad g_{33} = 1. \tag{12}$$

And by Eq. 8,

$$g_{11} = g_{22}. \tag{13}$$

Theorem I. For any given cylindrical coordinate system obtained directly from Eq. 1, g_{11} and g_{22} are equal and are the same as the metric coefficients obtained for the plane map, Eq. 2, while g_{33} is always equal to unity.

For the rotational system, Eq. 4,

$$g_{11} = g_{22} = \left(\frac{\partial \xi_1}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^1}\right)^2, \quad g_{33} = [\xi_1(u^1, u^2)]^2. \tag{14}$$

Again g_{11} and g_{22} are the same as obtained from the original plane system, Eq. 2. Similarly, for Eq. 5,

$$g_{11} = g_{22} = \left(\frac{\partial \xi_1}{\partial u^1}\right)^2 + \left(\frac{\partial \xi_2}{\partial u^1}\right)^2, \quad g_{33} = [\xi_3(u^1, u^2)]^2. \tag{15}$$

Theorem II. For any rotational coordinate system obtained directly from Eq. 1, g_{11} and g_{22} are equal and are the same as the metric coefficients obtained for the plane map, Eq. 2. The third coefficient g_{33} is always equal to the square of one of the original functions, Eq. 2.

It is sometimes convenient to modify a coordinate system obtained directly from the complex transformation, Eq. 1. The variable u can always be replaced, if desired, by a function of u, and v can be replaced by a function of v:

$$\begin{cases} u \to \rho(u), \\ v \to \sigma(v). \end{cases} \tag{16}$$

These transformations do not affect orthogonality or separability. In general, however, they alter the metric coefficients so that

$$g_{11} \neq g_{22}.$$

In the interests of simplicity, the present paper is limited to coordinate systems obtained directly from Eq. 1 and represented by Eqs. 3, 4, 5. Any of these systems, however, can be subsequently rewritten by use of transformation (16), and this change will not affect separability.

For example, take the transformation

$$x = e^w.$$

The map in the z-plane is represented by

$$\begin{cases} x = e^u \cos v, \\ y = e^u \sin v, \end{cases}$$

and the metric coefficients are $g_{11} = g_{22} = e^{2u}$. In the z-plane, u = const. represents a family of concentric circles about the origin, while v = const. indicates a set of radial lines. Cylindrical and rotational systems are obtained by use of Eqs. 3, 4, 5; and $g_{11} = g_{22}$ in all cases.

As an alternative, one may find it convenient to make the further transformation,

$$u = \ln r$$

or

$$\begin{cases} e^u = r, \ \xi_1 = r \cos \theta, \\ v = \theta, \ \xi_2 = r \sin \theta. \end{cases}$$

This gives the usual circular-cylinder coordinates,

$$\begin{cases} x = \xi_1 = r \cos \theta, \\ y = \xi_2 = r \sin \theta, \\ z = z \end{cases}$$

and the spherical coordinates, Eq. 5,

$$\begin{cases} x = \xi_3 \cos \psi = r \sin \theta \cos \psi, \\ y = \xi_3 \sin \psi = r \sin \theta \sin \psi, \\ z = \xi_1 = r \cos \theta. \end{cases}$$

But in these coordinates, g_{11} and g_{22} are no longer equal:

$$g_{11} = 1, \ g_{22} = r.$$

We shall now consider the necessary and sufficient conditions for the separation of the Helmholtz and Laplace equations. The general conditions have been formulated elsewhere (3), so it is now necessary merely to state the simplified conditions resulting from the use of our particular coordinates. Attention will be confined to the coordinate systems of Eqs. 3, 4, and 5 with $g_{11} = g_{22}$ and with the further restriction that the potential is independent of the third coordinate:

$$\varphi = \varphi(u^1, u^2).$$

2. *R*-SEPARABILITY OF THE HELMHOLTZ EQUATION

The Helmholtz equation is

$$\nabla^2\varphi + (k)^2\varphi = 0,$$

where k is a constant. The necessary and sufficient condition (3) for R-separability of the Helmholtz equation is

$$\sum_{i=1}^{n} \frac{1}{f_i g_{ii}} \frac{\partial}{\partial u_i}\left(f_i \frac{\partial R}{\partial u^i}\right) + \left[\frac{\alpha_1}{Q} - (k)^2\right] R = 0, \qquad (a)$$

where the functions Q and f_i are defined by

$$g_{ii} = \frac{S}{M_{i1}} Q(u^1, u^2, \cdots u^n), \qquad (b)$$

and

$$\frac{\sqrt{g}}{SQ} = \left[\prod_{i=1}^{n} f_i(u^i)\right] \cdot [R(u^1, u^2, \cdots u^n)]^2. \qquad (c)$$

Here S is the Stäckel determinant,

$$S = \begin{vmatrix} \Phi_{11}(u^1) & \Phi_{12}(u^1) \cdots \Phi_{1n}(u^1) \\ \Phi_{21}(u^2) & \Phi_{22}(u^2) \cdots \Phi_{2n}(u^2) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \Phi_{n1}(u^n) & \Phi_{n2}(u^n) \cdots \Phi_{nn}(u^n) \end{vmatrix}$$

and M_{i1} is the cofactor of Φ_{i1}. There are n separation constants α_1, $\alpha_2, \cdots \alpha_n$.

An outstanding feature of the Stäckel determinant is that all elements of the first row are functions of the first space coordinate u^1, all elements of the second row are functions of u^2, etc. The elements $\Phi_{ij}(u^i)$ of the determinant are arbitrary functions of u^i, though a random choice of functions does not in general lead to an orthogonal coordinate system in euclidean space. For a given coordinate system, characterized by a given set of metric coefficients g_{ii}, the functions $\Phi_{ij}(u^i)$ are determined so that Eq. b is satisfied. This procedure gives for circular-cylinder coordinates,

$$S = \begin{vmatrix} 0 & -\frac{1}{r^2} & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

and for spherical coordinates,

$$S = \begin{vmatrix} 1 & -\frac{1}{r^2} & 1 \\ 0 & 1 & -\frac{1}{\sin^2\theta} \\ 0 & 0 & 1 \end{vmatrix}.$$

In both of these examples, $Q(u^1, u^2, u^3)$ is 1. With some coordinate systems, however, it is impossible to set up a Stäckel determinant satisfying Eq. *b* unless Q is a function of the coordinates.

For the special cases considered in this paper, R-separability is obtained if the assumption of

$$\varphi = \frac{U^1(u^1) \cdot U^2(u^2)}{R(u^1, u^2)} \tag{17}$$

leads to the separation of the partial differential equation into two ordinary differential equations. Also, the Stäckel determinant is a function of only two variables:

$$S = \begin{vmatrix} \Phi_{11}(u^1) & \Phi_{12}(u^1) \\ \Phi_{21}(u^2) & \Phi_{22}(u^2) \end{vmatrix},$$

but $\sqrt{g}$ contains all three metric coefficients:

$$\sqrt{g} = \sqrt{g_{11}\, g_{22}\, g_{33}}.$$

Justification for these statements can be obtained from the previous paper (3).

Equation (*b*) now reduces to

$$g_{11} = g_{22} = \frac{SQ}{M_{11}} = \frac{SQ}{M_{21}}.$$

Thus $M_{11} = M_{21}$, or

$$\Phi_{22}(u^2) = -\Phi_{12}(u^1) = \text{const.}$$

Without loss of generality, we can set this constant equal to unity, so the Stäckel determinant becomes

$$S = \begin{vmatrix} \Phi_{11} & -1 \\ \Phi_{21} & 1 \end{vmatrix} = \Phi_{11}(u^1) + \Phi_{21}(u^2). \tag{18}$$

Theorem III. In any coordinate system obtained from Eq. 1, with $\varphi = \varphi(u^1, u^2)$, the Stäckel determinant is always equal to the sum of a function of u^1 only and a function of u^2 only, Eq. 18.

For the Helmholtz equation (3)

$$Q = 1 \quad \text{and} \quad \alpha_1 = (k)^2 + A,$$

where A is a constant. Thus from Eqs. *b* and 18, we obtain

$$g_{11} = g_{22} = S = \Phi_{11}(u^1) + \Phi_{21}(u^2). \tag{19}$$

From (*c*) and (*a*),

$$\sqrt{g_{33}} = f_1(u^1) \cdot f_2(u^2) \cdot (R)^2, \tag{20}$$

$$\sum_{i=1}^{3} \frac{1}{f_i} \frac{\partial}{\partial u^i} \left(f_i \frac{\partial R}{\partial u^i} \right) + ARS = 0. \tag{21}$$

Evidently, Eq. 20 is merely a definition of R and imposes no restriction on the coordinate system. The conditions for R-separability of the Helmholtz equation are therefore Eqs. 19 and 21.

For a cylindrical system, $\sqrt{g_{33}} = 1$, so Eq. 20 becomes

$$1 = f_1(u^1) \cdot f_2(u^2) \cdot (R)^2.$$

Without loss of generality, therefore, we can set

$$f_1 = f_2 = R = 1.$$

In other words, separability of the Helmholtz equation in cylindrical coordinates is obtainable only if $R = 1$. Indeed, this conclusion holds also for the more general case (3) where $\varphi = \varphi(u^1, u^2, u^3)$:

Theorem IV. R-separability of the Helmholtz equation is never possible for a cylindrical coordinate system.

We therefore turn to rotational coordinates. Here $\sqrt{g_{33}} = \xi$, where $\xi = \xi_1$ for the system of Eq. 4 and $\xi = \xi_2$ for the system of Eq. 5. Equation 20 becomes

$$\xi = f_1(u^1) \cdot f_2(u^2) \cdot (R)^2.$$

Thus,

$$\frac{\partial \xi}{\partial u^i} = f_2 \left[\frac{df_1}{du^1} (R)^2 + 2 R f_1 \frac{\partial R}{\partial u^1} \right]$$

or

$$f_1 \frac{\partial R}{\partial u^1} = \frac{1}{2} \left[\frac{1}{R f_2} \frac{\partial \xi}{\partial u^1} - R \frac{df_1}{du^1} \right].$$

Similarly,

$$f_2 \frac{\partial R}{\partial u^2} = \frac{1}{2} \left[\frac{1}{R f_1} \frac{\partial \xi}{\partial u^2} - R \frac{df_2}{du^2} \right].$$

Substitution in Eq. 21 gives

$$-\frac{1}{(R)^4 (f_1 f_2)^2} \left[\left(\frac{\partial \xi}{\partial u^1} \right)^2 + \left(\frac{\partial \xi}{\partial u^2} \right)^2 \right] + \left[\left(\frac{1}{f_1} \frac{df_1}{du^1} \right)^2 + \left(\frac{1}{f_2} \frac{df_2}{du^2} \right)^2 \right]$$
$$+ \frac{2}{(R)^2 f_1 f_2} \left[\frac{\partial^2 \xi}{(\partial u^1)^2} + \frac{\partial^2 \xi}{(\partial u^2)^2} \right] - 2 \left[\frac{1}{f_1} \frac{d^2 f_1}{(du^1)^2} + \frac{1}{f_2} \frac{d^2 f_2}{(du^2)^2} \right] + 4 AS = 0.$$

Utilizing Eqs. 7, 14, 19, 20, we obtain

$$\left(\frac{1}{f_1} \frac{df_1}{du^1} \right)^2 - \frac{2}{f_1} \frac{d^2 f_1}{(du^1)^2} + \left(\frac{1}{f_2} \frac{df_2}{du^2} \right)^2 - \frac{2}{f_2} \frac{d^2 f_2}{(du^2)^2} = \frac{g_{11}}{(\xi)^2} - 4 AS.$$

This equation may be written

$$\frac{g_{11}}{g_{33}} = \Omega_1(u^1) + \Omega_2(u^2), \tag{22}$$

where

$$\begin{cases} \Omega_1(u^1) = \left(\frac{1}{f_1}\frac{df_1}{du^1}\right)^2 - \frac{2}{f_1}\frac{d^2f_1}{(du^1)^2} + 4A\Phi_{11}, \\ \Omega_2(u^2) = \left(\frac{1}{f_2}\frac{df_2}{du^2}\right)^2 - \frac{2}{f_2}\frac{d^2f_2}{(du^2)^2} + 4A\Phi_{21}. \end{cases} \tag{23}$$

Theorem V. The necessary and sufficient conditions for R-separability of the Helmholtz equation, in orthogonal rotational coordinates in euclidean 3-space with $\varphi = \varphi(u^1, u^2)$, are that g_{11} and g_{11}/g_{33} be expressible as sums of separated functions:

$$\begin{cases} g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2), & (19) \\ \frac{g_{11}}{g_{33}} = \Omega_1(u^1) + \Omega_2(u^2), & (22) \end{cases}$$

where Ω_i are defined by Eq. 23.

3. *R*-SEPARABILITY OF THE LAPLACE EQUATION

Separability conditions for the Laplace equation,

$$\nabla^2\varphi = 0,$$

may be obtained *ab initio* or they may be taken as a special case of conditions for the Helmholtz equation with $k = 0$. Corresponding to Theorem IV, we have

Theorem VI. R-separability of the Laplace equation is never possible for a cylindrical coordinate system.

For Laplace's equation in rotational coordinates, $k = 0$, and Eqs. 22 and 23 apply with $A = \alpha_1$. Note that the restrictions are less severe with the Laplace equation than with the Helmholtz, since Q in Eq. (*b*) is not necessarily a constant but may be entirely arbitrary.

Theorem VII. The necessary and sufficient condition for R-separability of the Laplace equation, in orthogonal rotational coordinates in euclidean 3-space with $\varphi = \varphi(u^1, u^2)$, is that g_{11}/g_{33} be expressible as a sum of separated functions:

$$\frac{g_{11}}{g_{33}} = \Omega_1(u^1) + \Omega_2(u^2), \tag{22}$$

where Ω_i are defined by Eq. 23 and $A = \alpha_1$.

4. SIMPLE SEPARABILITY OF THE HELMHOLTZ EQUATION

For simple separability, $R = 1$ and Eq. 20 becomes

$$\sqrt{g_{33}} = f_1(u^1) \cdot f_2(u^2). \tag{20a}$$

Also, Eq. a reduces to

$$[\alpha_1 - (k)^2 Q]\, S = 0.$$

But since $S \neq 0$ and $Q = 1$,

$$\alpha_1 = (k)^2. \tag{21a}$$

And since $Q = 1$,

$$g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2). \tag{19}$$

With cylindrical coordinates, $\sqrt{g_{33}} = 1$, so Eq. 20a is always satisfied by

$$f_1 = f_2 = 1. \tag{20b}$$

Equation 21a evaluates the first separation constant; and neither (21a) nor (21b) imposes any restriction on the coordinate system. So the sole condition for simple separability of the Helmholtz equation in cylindrical coordinates is Eq. 19. With rotational coordinates, Eq. 20a still imposes a restriction:

$$\sqrt{g_{33}} = \xi = f_1 f_2 \tag{20a}$$

and Eq. 19 also applies.

Theorem VIII. The necessary and sufficient condition for simple separability of the Helmholtz equation, in cylindrical coordinates obtained from Eq. 1 and with $\varphi = \varphi(u^1, u^2)$, is that g_{11} be expressible as a sum:

$$g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2). \tag{19}$$

Theorem IX. The necessary and sufficient conditions for simple separability of the Helmholtz equation, in any rotational coordinate system obtained from Eq. 1 and with $\varphi = \varphi(u^1, u^2)$, are given by Eqs. 19 and 20a.

5. SIMPLE SEPARABILITY OF THE LAPLACE EQUATION

For simple separability of the Laplace equation, $R = 1$ and $k = 0$. Returning to Eqs. a, b, and c, we find

$$g_{11} = SQ, \tag{19a}$$

$$g_{33} = f_1(u^1) \cdot f_2(u^2), \tag{20a}$$

and

$$\alpha_1 = 0. \tag{21b}$$

With cylindrical coordinates, we may write for Eq. 20a,

$$f_1 = f_2 = 1. \tag{20b}$$

Since Eqs. 19a and 21b impose no restriction on the coordinates, we conclude that the Laplace equation separates in an infinite number of cylindrical coordinate systems:

Theorem X. The Laplace equation is simply separable in every cylindrical coordinate system obtained from Eq. 1 with $\varphi = \varphi(u^1, u^2)$.

With rotational systems, $\sqrt{g_{33}} = \xi$ and the sole requirement for separability is that g_{33} be a product:

$$\sqrt{g_{33}} = \xi = f_1(u^1) \cdot f_2(u^2). \tag{20a}$$

Theorem XI. The necessary and sufficient condition for simple separability of the Laplace equation, in any rotational coordinate system obtained from Eq. 1 with $\varphi = \varphi(u^1, u^2)$, *is given by Eq. 20a.*

6. THE SEPARATION EQUATIONS

If either the Laplace or the Helmholtz equation separates, the separated equations are always (3)

$$\frac{1}{f_i}\frac{d}{du^i}\left(f_i \frac{dU^i}{du^i}\right) + U^i \sum_{j=1}^{n} \alpha_j \Phi_{ij} = 0, \tag{24}$$

where $i = 1, 2, 3, \cdots n$, and Φ_{ij} are the elements of the Stäckel determinant. For the coordinate systems considered in this paper, $n = 2$ and $\Phi_{21} = -1$, $\Phi_{22} = +1$, so the separated equations are

$$\begin{cases} \dfrac{1}{f_1}\dfrac{d}{du^1}\left(f_1 \dfrac{dU^1}{du^1}\right) + [\alpha_1\Phi_{11} - \alpha_2]\, U^1 = 0, \\ \dfrac{1}{f_2}\dfrac{d}{du^2}\left(f_2 \dfrac{dU^2}{du^2}\right) + [\alpha_1\Phi_{21} + \alpha_2]\, U^2 = 0. \end{cases} \tag{25}$$

If the coordinates are cylindrical, $f_1 = f_2 = 1$ and Eq. 25 becomes

$$\begin{cases} \dfrac{d^2U^1}{(du^1)^2} + [\alpha_1\Phi_{11} - \alpha_2]\, U^1 = 0, \\ \dfrac{d^2U^2}{(du^2)^2} + [\alpha_1\Phi_{21} + \alpha_2]\, U^2 = 0. \end{cases} \tag{25a}$$

7. BIPOLAR COORDINATES

As an example, take the transformation,[5]

$$\bar{z} = \frac{c(e^w + 1)}{e^w - 1}$$

where $\bar{z} = x - iy$ and $w = u + iv$. Thus,

$$x - iy = \frac{c[(e^u \cos v + 1) + i\, e^u \sin v]}{(e^u \cos v - 1) + ie^u \sin v}$$

$$= \frac{c[(e^{2u} - 1) - i\, 2\, e^u \sin v]}{e^{2u} - 2e^u \cos v + 1,}$$

[5] The transformation could equally well be written with z instead of its complex conjugate $\bar{z}$.

or

$$\begin{cases} x = \dfrac{c \sinh u}{\cosh u - \cos v}, \\ y = \dfrac{c \sin v}{\cosh u - \cos v}. \end{cases} \tag{26}$$

From Eq. 12,

$$g_{11} = g_{22} = \frac{c^2}{(\cosh u - \cos v)^2}.$$

From the foregoing plane-transformation, we now formulate a cylindrical coordinate system (u^1, u^2, u^3):

$$\begin{cases} x = \dfrac{c \sinh u^1}{\cosh u^1 - \cos u^2}, \\ y = \dfrac{c \sin u^2}{\cosh u^1 - \cos u^2}, \\ z = u^3. \end{cases} \tag{27}$$

The metric coefficients are

$$g_{11} = g_{22} = \frac{c}{(\cosh u^1 - \cos u^2)^2}, \qquad g_{33} = 1. \tag{28}$$

These equations represent *bipolar-cylinder coordinates*. The surfaces $u^1 = \text{const.}$ are eccentric circular cylinders about the polar lines $(y = 0, x = \pm c)$. The surfaces $u^2 = \text{const.}$ are circular cylinders with centers on the y-axis, while $u^3 = \text{const.}$ represents a family of parallel planes.

According to Section 4, the Helmholtz equation separates if and only if

$$g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2). \tag{19}$$

Evidently, Eq. 28 does not satisfy this condition, so *the Helmholtz equation does not separate in bipolar-cylinder coordinates*. The Laplace equation, however, separates in any of our cylindrical systems. This is simple separability, so $\alpha_1 = 0$ and Eq. 25*a* gives the separation equations:

$$\begin{cases} \dfrac{d^2U^1}{(du^1)^2} - \alpha_2 U^1 = 0, \\ \dfrac{d^2U^2}{(du^2)^2} + \alpha_2 U^1 = 0. \end{cases}$$

The solutions of these ordinary differential equations are exponentials and circular functions, and the final solution of a Dirichlet or a Neumann problem in bipolar-cylinder coordinates is obtained as a sum of terms of the form,

$$\varphi = U^1 \cdot U^2.$$

Turning now to the rotational coordinates obtained from the plane map, Eq. 26, we have two possibilities. Rotation of the map about its y-axis gives *toroidal coordinates:*

$$\begin{cases} x = \dfrac{c \sinh u^1 \cos \psi}{\cosh u^1 - \cos u^2}, \\ y = \dfrac{c \sinh u^1 \sin \psi}{\cosh u^1 - \cos u^2}, \\ z = \dfrac{c \sin u^2}{\cosh u^1 - \cos u^2}, \end{cases} \tag{29}$$

with

$$g_{11} = g_{22} = \frac{c^2}{(\cosh u^1 - \cos u^2)^2}, \qquad g_{33} = \frac{c^2 \sinh^2 u^1}{(\cosh u^1 - \cos u^2)^2}. \tag{30}$$

Rotation about the x-axis of the plane map, Eq. 26, gives *bispherical coordinates:*

$$\begin{cases} x = \dfrac{c \sin u^2 \cos \psi}{\cosh u^1 - \cos u^2}, \\ y = \dfrac{c \sin u^2 \sin \psi}{\cosh u^1 - \cos u^2}, \\ z = \dfrac{c \sinh u^1}{\cosh u^1 - \cos u^2}, \end{cases} \tag{31}$$

with

$$g_{11} = g_{22} = \frac{c^2}{(\cosh u^1 - \cos u^2)^2}$$

and

$$g_{33} = \frac{c^2 \sin^2 u^2}{(\cosh u^1 - \cos u^2)^2}. \tag{32}$$

Now consider separability in these rotational coordinates. According to Theorems V and VIII, the Helmholtz equation separates only if g_{11} is a separable sum:

$$g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2).$$

Since neither toroidal nor bispherical coordinates satisfy this condition, the Helmholtz equation does not separate (simple or R) in these coordinates.

For simple separability of the Laplace equation, the necessary and sufficient condition (Theorem XI) is

$$g_{33} = f_1(u^1) \cdot f_2(u^2),$$

which is not satisfied. We conclude that Laplace's equation is not simply separable in toroidal or bispherical coordinates.

The remaining possibility is R-separability of the Laplace equation. With toroidal coordinates,

$$\frac{g_{11}}{g_{33}} = \frac{1}{(\sinh u^1)^2},$$

which is a simple case of the required form, $\Omega_1(u^1) + \Omega_2(u^2)$. Also,

$$Q = \frac{c^2}{(\cosh u^1 - \cos u^2)^2},$$

$$R = \frac{1}{\sqrt{\cosh u^1 - \cos u^2}},$$

$$\Phi_{11} = 1, \quad \Phi_{21} = 0, \quad f_1 = c \sinh u^1, \quad f_2 = 1.$$

Substitution in Eq. 23 shows that this equation is also satisfied and $\alpha_1 = \frac{1}{4}$. Therefore the Laplace equation is R-separable in toroidal coordinates.

Similarly in bispherical coordinates,

$$\frac{g_{11}}{g_{33}} = \frac{1}{(\sin u^2)^2},$$

which satisfies Eq. 26. Also,

$$Q = \frac{c^2}{(\cosh u^1 - \cos u^2)^2},$$

$$R = \frac{1}{\sqrt{\cosh u^1 - \cos u^2}},$$

$$\Phi_{11} = 0, \quad \Phi_{21} = 1, \quad f_1 = 1, \quad f_2 = c \sin u^2.$$

Equation 23 is again satisfied and $\alpha_1 = -\frac{1}{4}$, so Laplace's equation is R-separable in bispherical coordinates.

8. ONE-DIMENSIONAL SOLUTIONS

The previous part of the paper has dealt with solutions in which the potential is a function of two variables:

$$\varphi = \varphi(u^1, u^2).$$

We now consider one-dimensional solutions, $\varphi = \varphi(u^i)$, of the Helmholtz and Laplace equations. Since in this special case there is no separation of variables, one might suppose that solutions would be possible in all orthogonal coordinate systems. But such is far from being the case.

The Helmholtz equation in 3-space is

$$\frac{1}{\sqrt{g}} \sum_{i=1}^{3} \frac{\partial}{\partial u^i} \left(\frac{\sqrt{g}}{g_{ii}} \frac{\partial \varphi}{\partial u^i} \right) + (k)^2 \varphi = 0. \tag{33}$$

Assume that a solution exists in terms of a single coordinate, say u^i:

$$\varphi = \varphi(u^i).$$

Then two of the derivatives of Eq. 33 become zero, and the equation reduces to

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial u^i}\left(\frac{\sqrt{g}}{g_{ii}}\frac{d\varphi}{du^i}\right) + (k)^2\varphi = 0$$

or

$$\frac{d^2\varphi}{(du^i)^2} + \left[\frac{g_{ii}}{\sqrt{g}}\frac{\partial}{\partial u^i}\left(\frac{\sqrt{g}}{g_{ii}}\right)\right]\frac{d\varphi}{du^i} + (k)^2 g_{ii}\varphi = 0. \tag{34}$$

A one-dimensional solution is possible only if the coefficients in Eq. 34 are constants or functions of u^i only. For the second term of Eq. 34, it is necessary that

$$\frac{\sqrt{g}}{g_{ii}} = f_i(u^i)\cdot F_i, \tag{35}$$

where F_i is a function of the other two variables (not u^i). For the third term,

$$g_{ii} = g_{ii}(u^i). \tag{36}$$

Theorem XII. The necessary and sufficient conditions that the Helmholtz equation have a solution $\varphi = \varphi(u^i)$ are

$$\begin{cases} \dfrac{\sqrt{g}}{g_{ii}} = f_i(u^i)\cdot F_i, & (35) \\ g_{ii} = g_{ii}(u^i). & (36) \end{cases}$$

Theorem XIII. The necessary and sufficient condition that the Laplace equation have a solution $\varphi = \varphi(u^i)$ is

$$\frac{\sqrt{g}}{g_{ii}} = f_i(u^i)\cdot F_i. \tag{35}$$

The requirements of Theorem XII are hardly ever satisfied. Examination of the eleven coordinate systems of Eisenhart (2) shows that only in rectangular coordinates are Eqs. 35 and 36 completely satisfied. Beyond this one coordinate system, there exist only a few isolated one-dimensional solutions of the Helmholtz equation (for instance, in spherical coordinates $\varphi = \varphi(r)$ but not $\varphi = \varphi(\theta)$ or $\varphi = \varphi(\psi)$).

The Laplace equation is more fortunate. In fact, it is easily shown that an infinite number of coordinate systems satisfy Theorem XIII:

Theorem XIV. Laplace's equation allows one-dimensional solutions in every orthogonal cylindrical coordinate system.

TABLE I.—*Separability Conditions.*

For coordinate systems obtained from Eq. 1 with $\varphi = \varphi(u^1, u^2)$.

Equation	Type of Separation	Cylindrical Coordinates	Rotational Coordinates
Helmholtz	R	No separation	$\begin{cases} g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2) \\ \dfrac{g_{11}}{g_{22}} = \Omega_1(u^1) + \Omega_2(u^2)^* \end{cases}$
Laplace	R	No separation	$\dfrac{g_{11}}{g_{22}} = \Omega_1(u^1) + \Omega_2(u^2)^*$
Helmholtz	Simple	$g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2)$	$\begin{cases} g_{11} = \Phi_{11}(u^1) + \Phi_{21}(u^2) \\ \sqrt{g_{22}} = \xi = f_1(u^1) \cdot f_2(u^2) \end{cases}$
Laplace	Simple	Always separable	$\sqrt{g_{22}} = \xi = f_1(u^1) \cdot f_2(u^2)$

* Where $\Omega_i(u^i) = \left(\frac{1}{f_i}\frac{df_i}{du^i}\right)^2 - \frac{2}{f_i}\frac{d^2f_i}{(du^i)^2} + 4A\Phi_{i1}$.

REFERENCES

(1) W. E. BYERLY, "Fourier Series," Boston, Ginn & Co., 1893.
(2) L. P. EISENHART, "Separable Systems of Stäckel," *Ann. Math.*, Vol. 35, p. 284 (1934).
(3) P. MOON AND D. E. SPENCER, "Separability Conditions for the Laplace and Helmholtz Equations," JOUR. FRANKLIN INST., Vol. 253, p. 585 (1952).
(4) M. BÔCHER, "Reihenentwickelungen der Potentialtheorie," Berlin, B. G. Teubner, 1894.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

HAZARDS FROM STATIC ELECTRICITY

Electrostatics, the study of the phenomena of electrically charged particles or bodies, is of fundamental importance to science because many of the basic particles are charged and coulomb forces determine many of the properties of matter. Electrostatic forces are also of increasing industrial importance and have been applied to such processes as ore-separation and large-scale spray-painting. Painters, for instance, have found that a tremendous saving is achieved if the object to be painted is charged to several thousand volts.

In other industries, such as air conditioning, the manufacture of explosives, grain and flour storage, textiles, and printing, static electricity, under many circumstances, can be a distinct hazard. In a granary, for instance, the air is usually thick with fine flammable grain-dust particles. Grain sliding down a metal chute can result in an accumulation of static charge that, when it discharges in an electric spark, can easily ignite the fine dust particles and result in a dangerous and expensive fire.

For many years, the National Bureau of Standards has been consulted by other government agencies on problems arising from the hazards associated with static electricity. To meet a growing demand for this information, the Bureau is presently enlarging its research and standardization program, under the direction of F. L. Hermach, for establishing and evaluating methods of measurement and for determining the properties of materials and equipment used to reduce the hazard. The results of one important phase of the program, an investigation of the conductivity of floor coverings, are being utilized to reduce static electricity hazards in hospitals currently under construction and in Federal munition plants.

Specifications for conductive floors and other equipment depend on a determination of the safe limiting resistances between objects. NBS has participated in determining these limits, evaluating various safety factors, and devising suitable methods applicable to field as well as to laboratory testing of flooring and other materials, and is testing and evaluating various types of conductive floorings, coatings, and waxes. This work is conducted only for Federal and other government agencies.

A conductive floor need be only a moderately good conductor of static electricity—a resistance of one million ohms or less is ample for

* Communicated by the Director.

this purpose. Such a flooring provides a safe path over which separated electrostatic charges may be reunited as quickly as they were separated. It is this natural inclination of the charges to rapidly reunite that causes the hazard, creates the spark, or provokes the shock.

To determine the resistance characteristics of flooring, a sample is subjected to different voltages under varying conditions of humidity and temperature. The measuring procedure is also considered in the evaluation. The flooring resistance is measured with a special resistance-bridge as a function of pertinent variables such as voltage, humidity, and the type of electrode used in making the measurement. One test on a section of terrazzo flooring showed that for a variation of from 30 to 500 volts, the resistance of the sample decreased from about 4.5 to 1.5 megohms. A low resistance flooring—conductive linoleum—changed from almost 3 megohms at 40 volts to less than 10,000 ohms at only 150 volts.

An accompanying phase of these experiments is a determination of the actual safe upper limit of resistance. For example, tests show that voltages as high as 5000 volts can be developed by such common-everyday-occurrences as getting up from a plastic-covered chair. Oscillographic records, however, indicate that if the resistance between the person and the chair is less than 20 megohms, the voltages will not exceed 300 volts. This is less than the minimum sparking voltage in air and is thus not hazardous.

The magnitude of the electrostatic charges produced by separating objects depends on the material involved, their intimacy of contact, and their surface resistivities. However, many accidents do not arise from these direct effects but rather from the secondary effects resulting when the static electric charges are reunited. For instance, the electric shock experienced when a charged person touches a conducting object, while generally merely annoying, can cause serious damage if the person is precariously balanced on top of a ladder at the time he receives the minor shock. Also, the energies in electrostatic sparks may be large enough to ignite flammable gases or vapors that may be present in an enclosed room. Likewise, the "precipitation static" caused by corona discharges from highly charged aircraft in flight has been found to be often responsible for very serious interruptions to radio communications.

The most serious hazards, those of possible ignition of flammable gases or dusts, can best be mitigated by reducing the flammability or eliminating exposure to the flammable agent. This also reduces the hazard from other sources of ignition, such as arcs from the normal operation of electric equipment or the presence of hot bodies in the hazardous area. These measures, when coupled with the exclusion from the area of materials such as plastics, wool, and rubber, which have high electrical resistivity and "generate" high electrostatic charges, are often sufficient to reduce the hazard to a negligible value.

More elaborate methods of mitigation depend upon reuniting the charges as fast as they are separated in order to keep the voltage across the affected objects low. This can be accomplished by connecting stationary metallic objects to a common ground, by humidification (which provides a film of moisture of moderate conductivity on most objects), or by installing a conductive floor to provide electrical contact with the objects that move or rest upon it. The latter is generally considered the safest method where persons or moving objects may separate charges. Thus, in hospital operating rooms, for instance, the personnel normally wear shoes with conductive rubber soles, and the floors are made of conductive material. The separated charges developed in a person by walking, rising from a chair, or removing sheeting are then quickly reunited through the flooring before they can be transferred to another person or object.

In addition to the consultatory and testing services to other government agencies, members of the NBS staff have also served on a number of committees that have been organized to formulate rules and procedures governing static electricity preventive practices. In 1942, the Bureau published a circular[1] that outlines the problems and suggests solutions that are as timely today as they were 10 years ago. The circular discusses the nature and the origin of the charges of static electricity arising in industrial processes, and explains the various methods of mitigating the hazards that they introduce. It also gives the quantitative basis for an engineering treatment of the phenomena.

NBS PRE-CURED TAPE RESISTOR

The adhesive-tape resistor developed by the National Bureau of Standards has aroused wide interest since its announcement in 1951.[2] In the NBS tape-resistor system, designed primarily for electronic printed-circuit applications, small pieces of adhesive resistance-coated tape are simply pressed into place against metallic terminals at the proper points in the circuit. The resistor was developed as part of a program of miniaturization of airborne equipment sponsored by the Navy Bureau of Aeronautics. Despite its advantages, the method has been limited in some applications by the necessity for baking the supporting base material to cure the resistors after they have been pressed in place.

A new pre-cured wire-lead version of the tape resistor, now being made at NBS, obviates the need for heat-curing after placement in the circuit. The new resistors are made by pressing uncured resistor tape

[1] NBS Circular 438, "Static Electricity," available from the Superintendent of Documents, Washington 25, D. C., at 10 cents a copy.

[2] "A High-Temperature Adhesive Tape Resistor," *NBS Tech. News Bull.*, Vol. 35, p. 100 (July 1951). Described in detail in "An Adhesive Tape Resistor System," NBS Circular 530 Government Printing Office, Washington 25, D. C., 30 cents.

against both sides of suitable wire or metal-ribbon leads; the leads are thus sandwiched between two pieces of resistor tape. These units are then given the usual heat cure, which bonds the resistor tape to the leads and results in resistors that may be soldered or spot-welded into the circuit.

With the new pre-cured variation in addition to the basic press-on form, the range of possible applications of the NBS tape resistor is greatly extended. Characteristic advantages of the NBS tape resistor—compactness, stability, high-temperature operation—are largely retained in the pre-cured wire-lead design. Furthermore, the new resistor might well prove more economical to manufacture in quantity than other types having less desirable characteristics.

The basic NBS tape resistor is made by coating asbestos-paper tape with a mixture of carbon black or graphite, silicone resin, and solvent. Resistor dimensions are standardized at $\frac{1}{2}$ in. long and about $\frac{1}{8}$ in. wide; a variety of coating formulations have been developed to give a wide range of resistor values.

Leads for the pre-cured resistor are now being made from ribbon of thin silver or silver-plated copper at NBS. Leads extending $\frac{1}{2}$ in. beyond the resistor proper are used, bringing the over-all length to $1\frac{1}{2}$ in. Thickness is held to about 0.012 to 0.015 in.

Preliminary tests indicate that the pre-cured NBS tape resistor, when supported in air by its leads alone, will not provide the full dissipation of 0.25 watt at 200 C. for which the basic resistor was designed. Further test work is now in progress, and a suitable derating curve will be worked out.

PURIFICATION OF ZIRCONIUM

A rapid, convenient method for freeing zirconium of common impurities has recently been developed by W. S. Clabaugh and Raleigh Gilchrist of the National Bureau of Standards. The zirconium is obtained as the sulfate in good yield and very high purity. The method also provides a new technique for preparing zirconium oxide which may find application in commercial preparation of the metal.

Because of the complex nature of zirconium salts and the incomplete knowledge of their chemistry, the purification of zirconium has until now been a difficult matter. As a result, most laboratories have had to use zirconium in whatever purity they were able to purchase it. The NBS method, however, makes it possible for each laboratory to obtain zirconium free of all common impurities except hafnium.

It was found at the Bureau that when concentrated sulfuric acid is poured into a fairly concentrated aqueous solution of zirconium sulfate or chloride, a dense white crystalline precipitate of $Zr(SO_4)_2 \cdot 4H_2O$ is formed. This compound is soluble in an equal weight of water. By

dissolving the precipitate in water and reprecipitating the hydrated sulfate several times, zirconium sulfate of high purity is obtained.

For best results, one volume of concentrated sulfuric acid is added to two volumes of concentrated zirconium solution. The zirconium sulfate precipitate is caught on a sintered glass filter of medium or coarse porosity and washed with acid and acetone solutions. A small amount of hydrochloric acid is added during recrystallization to prevent the precipitation of any iron that may be present as an impurity.

The solution used to wash the precipitate consists of 75 volumes of water, 40 volumes of concentrated sulfuric acid, and 5 volumes of concentrated hydrochloric acid. After several washings with this mixed acid, three washings with acetone are recommended. Alcohol, however, cannot be used for this final washing because in some way it causes interference in subsequent recrystallizations, probably through complex formation.

In a recent test run at NBS, the method was applied to 1135 grams of commercial zirconium chloride containing about 0.3 per cent of iron and 2.7 per cent of hafnium. The zirconium chloride was dissolved in water to make 2.1 liters of solution, 1 liter of sulfuric acid was added, and the resulting product was recrystallized five times. A 70-per cent yield (1212 grams of zirconium sulfate) of high purity was obtained. Chemical analysis of the final product showed that it contained less than 0.1 part per million of iron and less than 0.1 part per million of copper. Spectrochemical methods gave a silver content of less than 1 part per million and less than 10 parts per million each of calcium, magnesium, sodium, and silicon. No other elements except hafnium were detected spectrochemically. A trace (about 0.01 per cent) of chloride ion was present, but this could be removed merely by recrystallizing the zirconium sulfate from water containing no hydrochloric acid after the iron had been eliminated.

The final product formed upon ignition of the hydrated sulfate was examined by X-ray diffraction and found to be the monoclinic form of ZrO_2. Thus, the NBS procedure also provides a convenient method for obtaining the oxide.

THE FRANKLIN INSTITUTE

MUSEUM

During the month of September visitors to the Museum will have their first opportunity to see the stamp collection of the late Harry L. Jefferys, which he bequeathed to the Institute. Mr. Jefferys was a true philatelist. That is to say, he was an investigator as well as a collector of stamps. Consequently he was never content with the acquisition of a single specimen, but sought as many as he could of the early issues in order to make comparisons which would bring to light varieties or deviations from the original. His collection was loaded with duplicates. By permission of the court the Institute was authorized to dispose of these duplicates, and the proceeds have been set aside to provide for the cost of keeping the collection up to date, of acquiring additions to make it complete, and of exhibiting it to the public.

Because it was imperative that the collection be completely representative, the selection of duplicates had to be performed with scrupulous care, and this task was accomplished gratuitously by three experts under the general supervision of Mr. Philip Ward, Jr. The tedious task of mounting and lettering was begun by Mr. Donald Steele, who has completed the nineteenth century issues. These comprise more than two hundred sheets which will be shown on the First Floor of the Museum between August 30 and September 28 (inclusive) in order that members of the American Philatelic Society may see the unique collection during the time of their annual convention held this year in Philadelphia.

The Jefferys Collection is rich in rarities, several of its items being unique and, therefore, likely to attract much attention from experts. The Western covers are described as "wonderful," and many envious eyes will be cast upon other rarities, such as stamps with inverted centers. This collection, however, should not be regarded as an assembly of rarities. It is an excellent reference collection which will serve as an aid to those collectors who seek to identify variations in the early issues of American stamps. Mr. Jefferys' knowledge in this department of philately was so extensive and his patience so inexhaustible that he was one of the few men who have succeeded in reconstructing a plate, that is, of fixing the individual stamps in their correct positions as they came off the printing press.

In addition to introducing the Jefferys Collection to the public, this exhibition is designed to serve as a tribute to the first Postmaster General of the United States, Benjamin Franklin.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

MARTIN, THOMAS L., JR.: Climate Control Through Ionization.

NEWTON, GEORGE C., JR.: Compensation of Feed-back Control Systems Subject to Saturation.

POLNAUER, FREDERICK F.: Biomechanics—A New Approach to Music Education.

KRZYWOBLOCKI, M. Z.: On the Invariants in the Turbulence in Compressible Viscous Fluids.

MANDEVILLE, C. E., E. SHAPIRO, R. I. MENDENHALL, E. R. ZUCKER AND G. L. CONKLIN: Radiations from Zr^{97} and Nb^{97}.

HETENYI, M.: Application of Maclaurin Series to the Analysis of Beams in Bending.

SHEN, D. W. C. AND H. N. G. BROADBENT: Analysis of Partly Symmetrical Machines by Means of Unitary Transformation.

GROSS, ERIC T. B.: Unbalance of Untransposed Overhead Lines.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

EMERICK, ROBERT HENDERSON. Heating Design and Practice. 1951.

ASTRONOMY

GAMOW, GEORGE. The Creation of the Universe. 1952.
HECKELMANN, ADOLF. Praktische Vermessungskunde. 1951.
LOVELL, BERNARD AND CLEGG, J. A. Radio Astronomy. 1952.
NININGER, HARVEY HARLOW. Out of the Sky; an Introduction to Meteorites. 1952.

BIOLOGICAL CHEMISTRY

BALDWIN, ERNEST. Dynamic Aspects of Biochemistry. 1952.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Advances in Catalysis and Related Subjects. Vol. 4. 1952.
BAKER, JOHN WILLIAM. Hyperconjugation. 1952.
DAMERELL, VIVIAN RICHARD. A Course in College Chemistry. 1952.
DATTA, R. L. Adhesives. 1950.
DEMING, HORACE G. General Chemistry. Ed. 6. 1952.
HARKINS, WILLIAM DRAPER. The Physical Chemistry of Surface Films. 1952.
HILL, ARTHUR E. A Brief Laboratory Guide for Qualitative Analysis. 1926.
INSTITUT INTERNATIONAL DE CHIMIE SOLVAY. Le Mecanisme de l'Oxydation. 1950.
LATIMER, WENDELL M. The Oxidation States of the Elements and Their Potentials in Aqueous Solutions. Ed. 2. 1952.
LEICESTER, HENRY M[ARSHALL] AND KLICKSTEIN, HERBERT S. A Source Book in Chemistry, 1400–1900. 1952.
MANSKE, RICHARD; HELMUTH FRED AND HOLMES, HENRY LAVERGNE, ED. The Alkaloids. Vol. 2. 1952.
MILLS, JOHN. Within the Atom, a Popular View of Electrons and Quanta. 1922.
MOORE, F. J. A History of Chemistry. 1918.

DICTIONARIES

HORNER, JOSEPH G. Dictionary of Terms Used in the Theory and Practice of Mechanical Engineering. Ed. 7. 1952.

ELECTRICITY AND ELECTRICAL ENGINEERING

NIPHER, FRANCIS E. Electricity and Magnetism. 1895.
ZWIKKER, C., Ed. Fluorescent Lighting. 1952.

ENGINEERING

ESHBACH, OVID WALLACE, Ed. Handbook of Engineering Fundamentals. Ed. 2. 1952.
KOCH, JACOBUS, J. [et al.]. Strain Gauges. 1952.
NEUBER, HEINZ. Theory of Notch Stresses. 1946.
SOLBERG, HARRY L.; CROMER, ORVILLE C. AND SPALDING, ALBERT R. Elementary Heat Power. Ed. 2. 1952.

GEOLOGY

HOBBS, WILLIAM HERBERT. Earth Evolution and Its Facial Expression. 1921.

GRAPHIC ARTS

SMITH, WILLIAM JOSEPH; TURNER, E. L. AND HALLAM, C. D. Photo-engraving in Relief. Ed. 3. 1951.

MANUFACTURE

SEAMAN, ROBERT G. AND MERRILL, ARTHUR M., ED. Machinery and Equipment for Rubber and Plastics. Vol. 1. 1952.

TRAVERS, RALPH. How to Make Professional Moulds and Castings. Ed. 2. 1952.

U. S. ORDNANCE DEPT. The Manufacture of Optical Glass and of Optical Systems. 1921.

MATHEMATICS

CANTOR, GEORG. Contributions to the Founding of the Theory of Transfinite Numbers. 1915.

ESTERMANN, T. Introduction to Modern Prime Number Theory. 1952.

FINNEY, D. Probit Analysis. Ed. 2. 1952.

HEACOCK, FRANK A. Graphic Methods for Solving Problems. 1952.

KORN, GRANINO ARTHUR AND KORN, THERESA M. Electronic Analog Computers. 1952.

LOOMIS, ELIAS. Elements of Analytical Geometry and of the Differential and Integral Calculus. 1863.

LAPLACE, PIERRE SIMON. A Philosophical Essay on Probabilities. [1902]. c1951.

SCHROEDER, ROLAND. Praktische Einführung in die Nomographie. 1951.

MECHANICAL ENGINEERING

HOUGHTON, P. S. Gears: Spur, Helical, Bevel and Worm. 1952.

PULL, ERNEST, Workshop Practice. Ed. 9. 1952.

SEQUENZ, HEINRICH. Die Wicklungen Elektrischer Maschinen. Vol. 1. 1950.

METALLURGY

SMITHELLS, COLIN J. Tungsten. Ed. 3. 1952.

THEWS, EDMUND RICHARD. Metallurgie Verarbeitung. Vol. 2. 1951.

PHARMACY AND PUBLIC HEALTH

MERCK & CO. Merck Index of Chemicals and Drugs. Ed. 6. 1952.

PHYSICS

ALLIS, WILLIAM PHELPS AND HERLIN, MELVIN A. Thermodynamics and Statistical Mechanics. 1952.

BURHAP, E. H. S. The Auger Effect. 1952.

GROOT, SYBREN RUURDS DE. Thermodynamics of Irreversible Processes. 1951.

IBBETSON, WILLIAM JOHN. An Elementary Treatise on the Mathematical Theory of Perfectly Elastic Solids. 1887.

KOLLER, LEWIS RICHARD. Ultraviolet Radiation. 1952.

LAUE, MAX VON. Theory of Superconductivity. 1952.

LOCKYER, SIR JOSEPH NORMAN. Studies in Spectrum Analysis. Ed. 4. 1886.

MAXWELL, JAMES CLERK. Matter and Motion. [1877].

MØLLER, C. The Theory of Relativity. 1952.

RICHARDSON, E. G. Ultrasonic Physics. 1952.

SEELY, FRED B. AND ENSIGN, NEWTON. Analytical Mechanics. 1952.

WEBER, ROBERT L., WHITE, MARSH W., AND MANNING, KENNETH W. College Physics. Ed. 2. 1952.

SANITARY ENGINEERING

BABBITT, HAROLD EATON. Engineering in Public Health. 1952.

SCIENCE

HERING, DANIEL W. Foibles and Fallacies of Science. 1924.
WEYL, HERMANN. Symmetry. 1952.

SUGAR

FISCHER, WILLY. Das Zuckerwaren-Laboratorium Einrichtung Arbeitsvorgänge Rezepturen. [1951].

TEXTILES

CARBONIA, PAOLO. Silk. 1952.
SOCIETY OF DYERS AND COLOURISTS. Photochemistry in Relation to Textiles. 1950.

THE FRANKLIN INSTITUTE

exists today because of the faith and generosity of the men and women who for 128 years have given time and money to its support.

The Institute welcomes financial gifts and bequests and hopes that all those who desire to perpetuate its work will make the The Institute one of their beneficiaries.

FORM OF BEQUEST

*I give and bequeath to The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts, the sum of*_____

_____

BOOK REVIEWS

ELECTRONIC ANALOG COMPUTERS, by Granino A. Korn and Theresa M. Korn. 378 pages, diagrams and illustrations, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $7.00.

The subject of d-c. analog computers is treated very effectively by the authors who have used such devices and have perfected through experience ways and means of avoiding the common pitfalls inherent in their use. The book commences with an introduction to the subject, then guides the reader through chapters on Practical Setup Procedure, The Application of D-C. Analog Computers to Representative Practical Problems, Theory and Design of Linear Computing Elements, Operational Amplifiers and Networks, D-C. Amplifiers for Computer Applications, Multiplication and Function Generation, Auxiliary Circuits and Computer Operation, and The Design of Complete D-C. Analog-Computer Installations. An Appendix entitled Some Properties of Parallel-Feedback-Type Operational Amplifiers is followed by an excellent Bibliography and Author and Subject Index.

Those working with analog computers should benefit by reading this book, as well as those who have problems they feel might be solved by computers of this type. Actual procedures are explained in detail and step-by-step summaries are given after the discussions.

Computers of this type built around d-c. amplifiers and servo multipliers are capable of solving a large variety of problems. Their main use has been in solving sets of simultaneous differential equations which may have varying coefficients and even be non-linear. Provision may be made for initial conditions and the solution recorded for later study. Though considerable time is required for set-up, determination of scale factors and checking before the final solution is obtained, there are many advantages of using electron means instead of the desk-type computing machine. Once the problem is set up, for instance, the effect of a change in a coefficient may be seen at a glance merely by turning a knob. Such matters are presented by Korn and Korn in a manner that makes the reader feel some of the enthusiasm for the subject obviously shared by the authors.

Consideration is given to both advanced time-scale computers and those operating on a real time basis. Considerable attention is given to foolproof methods of establishing scale factors—a phase of the work the authors feel has caused the most errors by those using analog computers. The only place this reviewer can suggest any improvement in the text is in the subject of checking the computer prior to running the solution. A valuable means of checking block diagrams, scale factors and wiring is to replace the feedback capacitors of the d-c. amplifiers by the appropriate feedback resistors and after recording the output of each amplifier, compare the solution with that of the basic machine equations in which integrations have been replaced by sums.

Considerable progress has been made in the design of d-c. amplifiers in recent years and banks of them can be built using standard circuits or purchased from a number of manufacturers. With little more, the electronic engineer who has not yet entered this field may, with the use of Korns' book, find new adventure.

DONALD B. HOUGHTON

THEORY OF SUPERCONDUCTIVITY, by M. von Laue, translated by Lothar Meyer and William Band. 140 pages, diagrams, 16×24 cm. New York, Academic Press, Inc., 1952. Price, $4.00.

This book is an English translation by Lothar Meyer and William Band of the author's "Theorie der Supraleitung" first published in 1947. The aim of the author is to present those aspects of the electrodynamics of superconductors which derive from the basic equations originally proposed by F. and H. London in 1935 to explain their macroscopic behavior. It

coincides to a great extent in this respect with "Superfluids" by F. London. However, in addition to the different approach and emphasis accorded to various topics by Professor von Laue, the theory has been extended to include tensor and nonlinear generalizations of the London theory.

After a brief statement of the fundamental facts of superconductivity and a discussion of the current distribution in superconductors in parallel, the fundamental equations of the Maxwell London theory are presented and the next few chapters are devoted to the detailed solution of a large number of practical problems involving stationary fields and currents, many of which correspond to experiments that have been performed in the laboratory. Further refinements of the theory are then introduced and followed by a discussion of high frequency waves in superconductors, the thermodynamics of the superconductive transition in large and small bodies, and a brief discussion of the intermediate state. The final chapter of the book contains a possible nonlinear extension of the theory.

A great amount of material has been included within the 140 pages of this book and the treatment of many topics is necessarily brief. Professor von Laue has contributed a great deal to the electrodynamic theory of superconductivity and this book reflects his own outlook as developed over a period of time. As such, it is invaluable to those who are actively engaged in research in this field and who wish to broaden their own understanding. It serves also as a valuable introduction to the theory, but the present reviewer feels that a more satisfactory introduction is provided by London's "Superfluids." WILLIAM F. LOVE

IMPERFECTIONS IN NEARLY PERFECT CRYSTALS, edited by W. Shockley. 490 pages, diagrams, 15×22 cm. New York, John Wiley & Sons, Inc., 1952. Price, $7.50.

Herein are published the important parts of the papers and discussions presented at a conference on the subject of "Lattice Imperfections," which was sponsored by The National Research Council through The Committee on Solids. The book is composed of seventeen chapters, by twenty contributors, divided into four parts: I. On the Nature of Imperfections in Nearly Perfect Crystals; II. The Role of Imperfections in Deformation; III. Diffusion and Related Phenomena; and IV. On the Properties and Effects of External and Internal Surfaces of Crystals.

Crystal imperfections are of vital importance both in fundamental researches and in technological advances, particularly their occurrence in metals. There are generally considered to be six primary types of imperfections, but dislocations, comparative newcomers to the group, are regarded by most competent thinkers to be of the greatest importance in the deformation and fracture of metals under stress. At the same time no one has as yet proved a given deformation to be due to a particular dislocation process. Thus with dislocations being of prime importance and with dislocation theories and particularly the interpretations of these theories as varied, and sometimes as muddled as the processes they were called upon to explain, it is not surprising that the principal emphasis of the conference was on dislocations. There existed a genuine need to have a better understanding of the present state of our knowledge, an understanding of the limitations of the theories, and recognition of the lack of vital information. This book succeeds remarkably well in assuaging the need on all three counts.

The book is also remarkable on more specific counts. It is reasonably indexed, and the references which are generally included at the end of the papers, collectively provide a glossary of most of the important published work on the subject of crystal imperfections. The comments and discussions following the papers are always enlightening and very often useful in pointing out the pitfalls of the topic under discussion. The opening paper, a synthesis of the field of "Imperfections in Nearly Perfect Crystals," by Professor F. Seitz provides not only a sorely needed coordination of our present knowledge, but also an excellent introduction to the following work, particularly for those who are not already experts on the subject.

The Editorial Committee and the publishers are to be commended on bringing us a book which will prove to be a stimulating addition to the library of all workers in solid state physics. As Professor Smoluchowski remarks in the Foreword to the book ". . . it can be considered a major step towards a better understanding of the solid state." F. E. JAUMOT, JR.

COSMOLOGY, by H. Bondi. 179 pages, 15×22 cm. New York, Cambridge University Press, 1952. Price, $4.50.

Man's ideas with regard to the temporal, spatial and material characteristics of the Universe are ever changing. Now Herman Bondi of the Cambridge group of Gold, Lyttleton, Hoyle and Bondi has expressed the thoughts of his colleague Gold and himself in a skillfully written book, "Cosmology."

The author introduces the "perfect cosmological principle" which holds that the Universe is homogeneous in its large-scale features and from every point in the Universe there is a constancy of its physical laws in space and time. The Universe presents the same aspects to an observer from every point except for local irregularities. Thus the author introduces his subject.

In Part II the observational evidence on which the cosmologies are built is described and in Part III, the significant part of the book, a complete resumé of the cosmological models of our day is analyzed and weighed.

Recent work by the German astronomer, Behr, indicates that the velocity-distance relationship, the "Hubble law," is incorrect. When Behr made his necessary revisions the estimated age of the Universe, as determined by the observed expansion, doubled—going from 1.7 to 3.4 billion years. It is difficult to understand why Bondi did not include this in his discussion.

An interesting by-product of the cosmological principle with present day overtures is the continuous creation of matter in interstellar space to give use to the "steady state" condition. Within the past month attempts have been made with the Brookhaven protosynchroton to create matter. If man can do it why not Nature?

The consequences of this new theory will eventually be tested against observations. Observational cosmology might be the key to fit these theories into the present day matrix of physical theories.

I. M. LEVITT

AUTOMATIC AND MANUAL CONTROL, edited by A. Tustin. 584 pages, illustrations, 19×25 cm. New York, Academic Press, Inc., 1951. Price, $10.00.

This volume comprises the papers contributed to the conference on Automatic Control arranged under the auspices of the Department of Scientific and Industrial Research, with the support of the Institution of Mechanical Engineers; the conference took place at the College of Aeronautics, Cranfield, England, from 16 July to 21 July, 1951. In attendance at the conference were upwards of 300 engineers, scientists, and mathematicians from many fields of activity and from most of the leading industrial countries of the world. The conference covered a wide field, but the emphasis of the papers is on general concepts and on methods of analysis, rather than on specific applications.

"Automatic and Manual Control" is printed with the quality of a textbook. It is profusely illustrated with excellent diagrams and tables. This volume contains the opening address by the President, 36 technical papers, an author index and a subject index. Each paper is introduced with an abstract and ends with a discussion relative to the paper. Each paper is well referenced. The papers in general are written on the presumption that the reader has some prior knowledge of the theory of control systems. Some of the papers presume a knowledge of particular branches of mathematics that may present difficulties to students or general readers. Several papers make use of the calculus of finite differences and the theory of functions of a complex variable.

The main groups of topics are: (a) Educational problems; (b) general theory; (c) process control; (d) non-linear problems; (e) systems working on intermittent data, and step-by-step servos; (f) the human operator; and (g) particular applications, including analogue computers and other devices. Approximately equal emphasis is placed upon electrical and mechanical methods of control, and the examples of process control are thermal, electrical, hydraulic, electrolytic, and chemical in principle. Sixteen demonstrations were set up at the conference. Each is written up showing illustrations where pertinent.

"Automatic and Manual Control" is a useful reference for the analytical designer of control equipment. It is also a source of information for those who have some knowledge of

the principles of the subject, and who wish to bring their knowledge up-to-date or to extend it in various more specialized directions. The four-page subject index, the author index and the technical references make this volume handy to use for reference purposes.

A. DONALD HAY

AN INTRODUCTION TO THE THEORY OF DIFFERENTIAL EQUATIONS, by Walter Leighton. 174 pages, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $3.50.

The author introduces the student to the theory of differential equations in the first chapter by a logical analysis of several differential equations. Thence, he presents the use of definite integrals, the fundamental existence therorem, linear equations with constant coefficients, the method of successive approximations, oscillation theory, characteristic functions (eigen values, eigen numbers), and some applications.

A specific application, had it been used, of the operator D to the method of successive approximations would have displayed the effectiveness of using the operator in solving differential equations in this manner; however, this was accomplished (in a hidden way) by the solution of algebraic equations by successive approximations.

Although Dr. Leighton stresses such topics as the method of successive approximations and the treatment of the linear differential equation of second order, this reader found the chapters treating oscillation theory and characteristic functions especially appealing.

Examples are interspersed in the text, as are groups of exercises. Answers to these exercises have been included at the end of the book.

The subject matter is presented in a clear and readable manner so that the student will not find it dry and tiresome.

E. A. FASKO

ELECTRONICS FOR COMMUNICATION ENGINEERS, by John Markus and Vin Zeluff. 610 pages, diagrams, 22×28 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $10.00.

"Electronics for Communication Engineers" is a collection of 252 articles selected from *Electronics* magazine. Since most of the articles were published during the years 1949-1951, they embody recent thinking on the subject. However, outstanding articles of earlier years were selected to round out the material presented in each of the 16 sections.

The editors are to be congratulated for the way the book is organized. The table of contents, the index, the arrangement of the articles under main headings, and the alphabetical arrangement of these headings make the material easily accessible. These main topics are: amplifiers, antennas, audio, cathode-ray tubes, components, electronic music, filters, measurements, microwaves, oscillators, power supplies, propagation, pulses, receivers, transmission lines, and transmitters. Most of the articles have a practical slant but many of them include enough basic theory to enable the reader to understand the design procedure and extend it to meet his own requirements. Numerous graphs, charts and nomographs put the material in readily accessible form. Many circuit diagrams are included.

Articles that particularly interested me were those on electronic music, equipment for citizens band measurements, magnesium delay lines, and blower selection for forced-air cooled tubes. However, many other subjects are covered and the reader is sure to find ones that interest him.

E. A. MECHLER

BASIC ENGINEERING THERMODYNAMICS, by Vincent W. Young. 558 pages, illustrations, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $6.50.

Judging by some textbooks, technical authors frequently assume that the prose used in presenting their material is of secondary importance and that the significant fact is to get the technical information into the book no matter how. That may be true in advanced texts where the reader is well informed and can read from equation to equation with little need for the accompanying discussions. But in less advanced texts, written for the beginning student, it is very important to have the material presented in such a fashion that the student understands it. Not only must the subject matter be selected wisely, both as to the amount the student can absorb and to the appropriateness, but it must also be explained in clear, concise language, so that one doesn't have to read, reread and rearrange before he understands. It is therefore very fortunate that "Basic Engineering Thermodynamics" in general, and particu-

larly because of the intangibility of some of its fundamental concepts, is written in just this manner. In the preface the author, Mr. Young, former professor of Mechanical Engineering at Oklahoma Agricultural and Mechanical College, stresses the importance of proper selection of material, clarity and readability, stating that he has striven for these objectives and hopes that his book has fulfilled his aims. It has.

Having taught for a number of years, Mr. Young knows the more difficult parts of thermodynamics and the methods of making these parts understandable. He does this primarily by simple examples and illustrative problems and by good, clear explanations. This is the strong point of the book, for the material is not much different from conventional books on thermodynamics. Opening with the fundamental definitions and concepts, Mr. Young proceeds to discuss the first law of thermodynamics with the closed and open system, the reversible process and cycle, the second law of thermodynamics, and entropy. He next discusses the pure substance, the perfect gas and mixtures of gases and vapors, the steady flow of fluids in connection with turbines and later through pipes and ducts, gas and vapor systems and the reciprocating steam engine, refrigeration and irreversibility. He concludes with discussions of real gases, thermodynamics of combustion and the transmission of heat. The inclusion of this last subject is not too often treated in books on thermodynamics, and consequently makes this present one that much more informative. Since this is a text book for undergraduate study, frequent problems are included throughout the text and at the end of each chapter.

E. W. HAMMER, JR.

EXTRUSION OF PLASTICS, RUBBER, AND METALS, by H. R. Simonds, A. J. Weith, and William Schack. 454 pp., illustrations, 16×23 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

This book is written primarily for the plastics engineer, and describes the extrusion process exclusively. All phases of this process are discussed in detail, the economics, the equipment, materials, and the methods. Even the theoretical aspects of the process are treated from the engineer's viewpoint.

The chapters on processes and equipment are amply supplied with figures and illustrations, facilitating a proper understanding of the production problems involved. These chapters offer technical information that can be followed even by a novice in the field. The necessary preliminary treatments of plastics, the various products of the extrusion machines, and the shapes obtainable are described in detail. A chapter on applications of this method is surprising in the number and broad coverage of the products possible.

Several chapters are devoted to the extrusion of rubber, metals, and miscellaneous materials, showing the significant advances that have been made in the extrusion process.

It is difficult to select any specific area of this book for criticism. Each of the chapters describes comprehensively a phase of the extrusion process in sufficient detail, and clearly enough, to appeal to the plastics engineer.

S. MUCHNICK

BOOK NOTES

DU PONT. THE AUTOBIOGRAPHY OF AN AMERICAN ENTERPRISE. 138 pages, color plates, illustrations, 24×32 cm. New York, Charles Scribner's Sons, 1952. Not for sale.

E. I. du Pont de Nemours & Co., Inc., on the occasion of its 150th anniversary, has published this history of American enterprise from 1802 to 1952 for private distribution to employees and stockholders. Beautifully bound and profusely illustrated with color portraits of many of du Pont's presidents and with halftones and steel engravings, the book is a masterpiece of the printing art. The chronicle, divided into twelve sections arranged chronologically, constitutes a thumbnail history of American business, primarily as exemplified through the growth of the du Pont Company. Pictures and story sketch the social and economic patterns of 150 years of American life, and point up the various ways in which the company has served the needs of the American people. Of particular interest are the section on "The Modern Corporation (1902–1914)," in which mass production and planned research opened new fields for American business, and the section on "Global War (1939–1945)," which outlines the vast contributions of the company to our war effort. This reviewer recommends that JOURNAL readers borrow

a copy from someone fortunate enough to possess one—or come to the Institute Library where a copy is on file—and enjoy an hour or two of reminiscing about our American system of enterprise.

STRENGTH OF MATERIALS, by John W. Breneman. Second edition, 140 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $3.00.

Primarily a textbook for extension, Armed Forces, correspondence, and home-study courses, this second edition provides an elementary text on strength of materials, using no mathematics beyond trigonometry. The chapter on the properties of engineering materials has been deleted and its essential information is now included in tabular form; otherwise, the second edition represents mainly a refinement of the first, by selection of different problems and examples, and by expanding the discussion of beam deflections. It is a practicable text, requiring a minimum of background preparation, and making use of step-by-step solved examples.

CONTROLLERS FOR ELECTRIC MOTORS, by Henry Duvall James and Lewis Edwin Markle. Second edition, 426 pages, illustrations, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $7.00.

Principal changes in the second edition of this book—designed primarily for control-equipment engineers and maintenance men—are: the addition of a chapter on magnetic amplifiers and their applications; a more complete description of dynamo electric amplifiers; a chapter on magnetic clutches and their use with synchronous motors; a rewriting of the chapter on crane control, to include recent d-c. control and saturated-core reactors; and remote and supervisory control has been added. These changes bring up-to-date a worthwhile treatment of control equipment.

DICTIONARY OF ARCHITECTURE, by Henry H. Saylor. 190 pages, plates, 11×17 cm. New York, John Wiley & Sons, Inc., 1952. Price, $4.50.

Designed to meet the need for a handy volume in which to find the spelling, pronunciation and concise definition of terms encountered in the study and practice of architecture, this modest book should prove its worth to students of architecture. It does not cover the technical terms of all the sciences and arts connected with architecture, but only those which an architect would use in discussing plans. A practical, phonetic system of indicating pronunciation is a welcome relief from the usual diacritical markings.

PRINCIPLES OF CHEMISTRY, by Joel H. Hildebrand and Richard E. Powell. Sixth edition, 444 pages, diagrams, 15×22 cm. New York, The Macmillan Co., 1952. Price, $4.50 Also available bound with "Reference Book of Inorganic Chemistry," by Latimer and Hildebrand, at $7.50.

In preparing this sixth edition of a standard text, the authors have made few changes from the extensively-rewritten fifth edition. Minor alterations in style have been made in many chapters, but the main revisions are those which bring up-to-date the discussions of the structure of matter, the properties of atoms, chemical binding, chemical kinetics, equilibrium and oxidation.

DIFFERENTIAL EQUATIONS, by Robert C. Yates. 215 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $3.75.

The author has prepared a brief, practical text for engineering students, designed to give them the necessary mathematics for analysis and solution of problems leading to differential equations. The text covers first order equations, general linear ordinary equations with constant coefficients, special equations of a nonlinear character, numerical solutions, and solution of the Wave equation by separation of variables. Also included are discussions of Fourier series and their applications. A notable contribution is the unique presentation of problems: first, the problem is presented in physical terms; then it is analyzed for its mathematical counterpart; then the mathematical solution is determined. This should prove to be a useful and practical text for a first course in the subject.

A portable hot plastic spray unit designed by a master mechanic at the Long Beach Naval Shipyard greatly facilitates application of hot plastic ship-bottom paint. The unit consists of a 250-gal. steam-jacketed mixing kettle, air operated propeller-type agitator, and heavy-duty double action air operated pump. In actual test with the unit, four men painted 40 per cent of a destroyer hull in less time it took nine men, using the older spray method, to cover the same area of another hull.

Very-High-Frequency Transistors.—A significant advance in transistor research which for the first time points the way to very-high-frequency applications in television, FM radio and point-to-point radio communications was reported recently by the Radio Corporation of America.

Several developmental point-contact transistors were made to oscillate at frequencies well up in the 100-to-200 megacycle band and one reached a record high frequency of 225 megacycles per second, according to Dr. Charles B. Jolliffe, Vice President and Technical Director of RCA. Such frequencies include the range in which FM radio and television signals are broadcast. The highest frequency value previously achieved by transistors, according to published reports, has been 50 megacycles per second.

The transistor, still in the developmental stage, consists of a speck of germanium crystal and fine contact wires and is no greater in size than a kernel of corn. It can perform many of the functions of electron tubes.

Prior to recent RCA experiments, transistors have been regarded as limited to relatively low-frequency applications, Dr. Jolliffe said. The new development promises to extend the use of the tiny transistors in high-frequency devices and to new applications in television, FM radio, point-to-point radio communication and other electronic equipment for military and civilian use.

The successful development of transistors which oscillate in the very-high-frequency region was accomplished by B. N. Slade, transistor engineer of the RCA Tube Department, RCA Victor Division, Harrison, N. J. This experimental work was conducted as a phase of RCA's transistor research program which is coordinated at the David Sarnoff Research Center of RCA, Princeton, N. J.

Mr. Slade pointed out that the research theory which led to the development of VHF transistors now makes possible the design of transistors which incorporate particular operating characteristics for a given application.

Tests conducted at the RCA Tube Department's Harrison, N. J., transistor laboratories confirmed a definite correlation between the spacing of a transistor's contact points and its frequency response. Generally speaking, Mr. Slade said, the closer the spacing, the higher the frequency.

Further tests established that a transistor's frequency response and stability are also determined to a large degree by the resistivity of its germanium crystal.

RCA transistor engineers, he explained, developed different combinations of spacing and resistivity values which enable them to design experimental transistors having a range of operational characteristics. One such combination of values also resulted in a transistor which oscillated with good stability at a frequency of 225 megacycles per second.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

2-DESOXY-D-GLUCOSE AS AN INHIBITOR OF ANAEROBIC GLYCOLYSIS IN TUMOR TISSUE

BY

GLADYS E. WOODWARD AND FRANCIS B. CRAMER

It has recently been shown that 2-desoxy-D-glucose (2DG), acting as an analogue of glucose, competitively inhibits the fermentation (anaerobic metabolism) of glucose by whole yeast(1). Preliminary experiments have now shown that 2DG also inhibits the anaerobic metabolism of glucose by surviving tumor cells in tissue slices.

Fresh Walker 256 rat carcinoma was sliced to a thickness of 0.5 mm. in a Stadie-Riggs microtome. The slices were suspended in an isotonic mixture of Ringer's solution and sodium bicarbonate in a Warburg apparatus(2). Approximately the same amount of fresh tissue (about 20 mg. dry weight) was used in each flask. The rate of anaerobic glucose metabolism was determined by measurement of the CO_2 liberated from bicarbonate by lactic acid elaborated by the tumor cells, following Warburg's procedure.

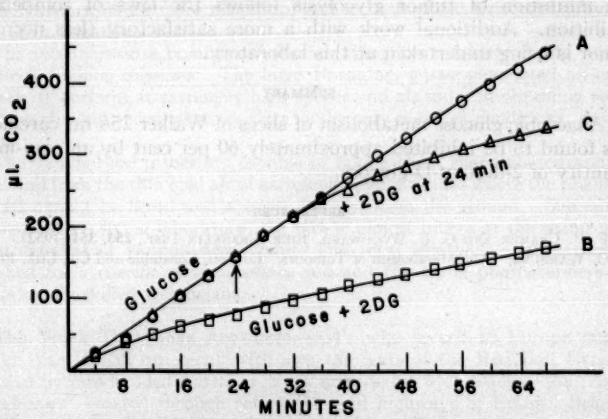

FIG. 1. Inhibitory effect of 0.05 M 2DG on glycolysis by Walker 256 tumor slices in bicarbonate-Ringer's medium containing 0.05 M glucose.

The experimental data obtained are presented graphically in Fig. 1. Curve A (control) represents the rate of CO_2 evolution in a bicarbonate-Ringer's solution containing 0.05 M glucose. Curve B shows the lowered rate of CO_2 evolution in the medium containing 0.05 M glucose when 0.05 M 2DG was also present before measurements were begun. Comparison of Curves A and B indicates that glucose metabolism was inhibited 63 per cent by an equi-molar quantity of 2DG.

Curve C represents the effect produced by the addition of an equi-molar quantity of 2DG to a system in active glycolysis, similar to the control experiment. After 24 minutes, sufficient 2DG was tipped in from the side-arm to establish the conditions of the experiment represented by Curve B. A period of about 20 minutes was required before the full inhibitory effect of the 2DG was established. Comparison of the rates of CO_2 evolution before and after addition of 2DG indicates 60 per cent inhibition of the anaerobic metabolism of glucose by an equi-molar quantity of 2DG. This is in good agreement with the value (63 per cent) obtained by comparison of Curves A and B. Additional experiments have confirmed these results.

The inhibitory effect of 2DG on glycolysis by Walker 256 tumor cells was slightly less than the inhibition (78 per cent) of the fermentation of glucose by whole yeast(1). The delayed response following addition of 2DG to the mixture in active glycolysis (Curve C) is probably due to the approximately 50-cell thickness of the tissue slices as contrasted with the immediate response to added 2DG in the individual-cell suspension of yeast. Insufficient data are available to determine whether the inhibition of tumor glycolysis follows the laws of competitive inhibition. Additional work with a more satisfactory (less necrotic) tumor is being undertaken at this laboratory.

SUMMARY

Anaerobic glucose metabolism of slices of Walker 256 rat carcinoma was found to be inhibited approximately 60 per cent by an equi-molar quantity of 2-desoxy-D-glucose.

REFERENCES

(1) F. B. CRAMER AND G. E. WOODWARD, JOUR. FRANKLIN INST., 253, 354 (1952).
(2) O. WARBURG, "The Metabolism of Tumours," London, Constable and Co., Ltd., 1930.

CURRENT TOPICS

Wide-Angle Photographic Lens.—Borne on the mighty wings of the Strategic Air Command's inter-continental reconnaissance bombers, American cameras "seeing" through unusually sharp wide-angle lenses can now record and bring back accurate information from any spot on the globe.

One of the important keys to the keen "vision" of the world-ranging RB-36 bombers of the 5th Strategic Reconnaissance Wing at Travis Air Force Base, Calif., Air Force officers disclosed, is a Bausch & Lomb photographic lens which enables a single aerial photograph to show three times as much area as was previously possible from the same altitude.

The lens, called the Metrogon, was developed by the Bausch & Lomb Optical Co. of Rochester, N. Y., and has been in use for some time but it has just been disclosed that Air Force developments now enable it to range the world in seeking out and recording vital ground information.

Ke to the importance of the coated Metrogon lens is its combination of very wide angle with sharpness and freedom from distortion at the relatively high speed of f/6.3. The Metrogon covers 90 deg. of field and has a focal length of 5¼ in. So clear is its definition that a photo from an altitude of a mile can show separate railroad ties anywhere within a two-mile radius beneath the plane, according to tests by the Bausch & Lomb Scientific Bureau. The distortion resulting from all photo lenses, especially those taking in a wide angle, has been almost completely eliminated in the Metrogon, thereby greatly increasing the fidelity of reconnaissance photos made through it.

In addition to Metrogons, the RB-36's cameras use two other Bausch & Lomb lenses, a 20-in. f/5.6 telephoto, and a 24-in. f/6.0 lens that provides 9-by-18-in. negatives.

The reconnaissance bombers of the 5th Wing are currently flying round-the-clock training missions. The huge 10-engine, jet-supplemented air-craft are able to perform at extremely high speeds and altitudes in obtaining aerial photographs and reconnaissance data under any weather conditions, day or night.

The Travis Base planes are capable of taking clear, sharp photographs in fine detail from the thin cold air of extremely high altitudes where the phantom aircraft cannot be heard and scarcely be seen from the ground. Sometimes referred to as a flying laboratory, the RB-36 almost bulges at the seams with aerial cameras of various sizes, shapes and focal lengths. Each camera is designed for a specific reconnaissance job and the aerial photographers are exceptionally skilled technicians.

The Truck That Goes Anywhere.—GI's who served in Europe during World War II still can recall with awe the feats of the Red Ball Express. Manned by sure-handed American truck drivers, big 6×6 trucks—the "Army Workhorses"—roared through the streets and highways of France, Belgium and Germany, hauling vital supplies to onrushing allied armies.

If ever we need the Red Ball again, we have trucks that are faster, easier to drive, easier to maintain, and carry larger payloads than their historic

predecessors. The new model "Workhorse" is GMC's brilliant M-135, a tactical vehicle that was evolved from all the experience obtained during the last war with more than a half-million GMC 6×6s.

The M-135 has GM's famed Hydra-Matic drive with eight speeds forward and two in reverse, and is powered by a new high-compression GMC 6-cylinder (302 cu. in.) gasoline engine developing 145 horsepower. The new model carries five tons hitting a top speed of 58 miles per hour, and can tow five tons, over, under or through almost any type of terrain.

With built-in sealing features and "snorkels," the GMC can travel submerged in water. On a test at Rainbow Springs, Fla., it bucked through dense jungle, knocking over trees up to four inches in diameter, and took to water like a fish. On winter maneuvers with 50 deg. below zero temperature, the M-135 again gave top performance.

Not even a 60 per cent grade can stop the mighty new GMC 6×6, whose six wheels include an automatic front wheel drive engagement which comes in when needed. With all these new features, the truck is one that GI drivers may count on to go virtually anywhere!

New Liquid-Level Control.—A new electrical system for sewage and water level control and indication has been announced by the General Electric Company's Control Department.

Called Thermasul, the new system can be used in wet wells, sumps, reservoirs, elevated tanks, standpipes, etc., and will control pump motors, pump motor speeds, and supervisory and alarm circuits.

Basically, the Thermasul element consists of a temperature-sensitive, sintered-sulfide compound around which is wound a resistance heater; the whole assembly hermetically sealed in a stainless-steel tube. In air, resistance of the element is low because the heater keeps the level detector approximately 280 degrees C. above the ambient air temperature. When the detector is immersed in liquid, however, the heat is conducted away and the resistance of the element jumps up rapidly, operating a small, remotely-located relay.

Thus, the Thermasul element acts as a switch, opening and closing the relay with the presence or absence of liquid. The relay, in turn, can be used to control pump motors, and circuits for alarm, supervisory, and control equipment. With the addition of standard accessories, Thermasul permits great versatility in pump programming.

According to G-E engineers, the non-corrodible construction and self-cleaning characteristics of the new device make it exceptionally useful for unattended locations. Deposits do not normally adhere to the detecting element because of the high operating temperature.

Thermasul systems can be mounted with simple pipe supports. Three conductor cable connections from level detectors to auxiliary control are the only electrical connections required with the exception of power-line pick-up. There are no moving parts to jam or bind, no electrodes to be shorted or otherwise fouled by deposits, and Thermasul is not adversely affected by greasy films or ice, the engineers said.

Both straight and hooked probes are available, the latter for use in low-level detection. Detectors are offered in lengths ranging from 1 to 10 ft; and an additional 10-ft. length of neoprene-covered cable is included.

A new magnetic material developed by the Naval Ordnance Laboratory, White Oak, Md., exhibits a coercive force of 3000 oersteds, greater than any known permanent magnetic material. Composed of bismuth and manganese, the new material has a flux density exceeded only by platinum-cobalt alloy; magnets can be made by powder metallurgy technique, thus insuring close tolerance of complicated shapes without machining. Production involves heating powdered manganese and bismuth at 700 C., hot-pressing the resulting bismuthide at 300° C. in a strong magnetic field. Technical evaluation, with respect to stability, aging, and shock is not yet available.

Russian-English Scientific Dictionary.—A series of studies which may eventually lead to the compilation of a new, comprehensive Russian-English scientific dictionary are being undertaken by Columbia University of NewYork under contract with the National Science Foundation.

During and since World War II the Russian scientific and technical vocabulary has expanded considerably and English speaking scientists have increasingly felt the need for more adequate specialized dictionaries than are now available. Support of the Columbia project is one phase of a larger program of the Foundation to facilitate the translation and distribution of Russian scientific information among scientists in the United States.

The program will include a systematic survey of existing Russian-English and English-Russian scientific and technical dictionaries to evaluate their adequacy in various scientific fields. Methods for compiling lists of words for which definitions are required and for obtaining scientific and linguistic accuracy in defining them will be developed and tested. The project will test these methods through preparation of special manuscripts in the fields of mathematics and metallurgy. It will also preface a comprehensive plan for compiling a full-scale dictionary, covering all fields of science, including estimates as to size and scope of the project, recommended procedures, staff requirements and costs.

Important New Sun Data Expected Soon.—In a short time, astronomers, physicists, and other scientists will have the most detailed data yet available on important aspects of solar radiation, according to Dr. John W. Evans of the High Altitude observatory at Climax, Colo. Dr. Evans directed activities of an expedition of astronomers who photographed an eclipse of the sun in Khartoum, Sudan, last Feb. 25. The expedition was sponsored by the Naval Research Laboratory, Washington.

Findings made as a result of the eclipse photographs may well have an important bearing on long-range weather forecastings, communications, and other fields related to the sun's effects on the earth, according to Dr. Evans.

At Khartoum, using specially-developed spectrographic cameras, the Naval Research laboratory astronomers photographed the eclipse at 4/10-second intervals in exposures 3/10 of a second long.

Photographs were made through a spectrograph, the high-precision optical instrument which is capable of identifying elements in amounts as small as one part in 100 million.

Two of these spectrographs were equipped with diffraction gratings, manufactured especially for the expedition by Bausch & Lomb, to split light rays

into spectrum according to wavelength. The surface of each of the 6-in.-wide gratings was ruled with 135,000 lines, all equally deep, equally spaced, and precisely parallel. "Their resolving power, or ability to separate light into distinct spectral lines approaches theoretical perfection," according to David Richardson, head of the spectroscopic research department of Bausch & Lomb Optical Co.

In use during the eclipse, the grating functioned perfectly, Dr. Evans said. "Through their use," he said, "we obtained the most detailed picture yet made of the height variations of the spectrum of the chromosphere." The chromosphere is the usually invisible layer that surrounds the sun above the visible atmosphere. Its height is estimated as above 12–14,000 kilometers.

The high degree of detail in the photographs obtained was such that instruments used in studying them have had to be re-calibrated, according to Dr. Evans. This task has just been completed in preparation for the months of study that will be needed to thoroughly evaluate the photographs. It is anticipated that additional evidence will be obtained as to the temperatures at various levels in the chromosphere—a measurement about which there is conflicting data and no theoretical explanation.

Electromagnetic Pump for Moving Molten Metals.—A new a-c. electromagnetic pump with no moving parts, for pumping liquid metals and other low resistance conducting fluids at temperatures up to 1000° F., has been announced by the Special Products Section of the General Electric Company.

According to engineers of the company's General Engineering Laboratory who developed it, the revolutionary new pump can be used to move any conducting fluid that has a specific resistance equal to or less than stainless-steel (approximately 90 microhm-cm). Liquid metals such as sodium, sodium-potassium alloys, and lithium, they said, fit this classification.

The new pump operates on the principle that a current-carrying conductor in a magnetic field is acted on by a force. In this case, the fluid serves as the conductor, so that when current and flux are properly applied, pumping occurs.

One of the three basic elements of the pump, a transformer unit, sets up a high current through the liquid metal and, simultaneously, a magnetic field perpendicular to that current. A cell unit, which consists of a flattened stainless-steel pipe, carries the liquid metal through the magnetic field. The third basic part, a capacitor unit, provides power factor correction.

The amount of liquid metal which can be pumped in a given period varies with the pressure. At 20 psi. the pump is able to move approximately 28 gal. of molten metal per minute, the engineers said.

The transformer unit of the new electromagnetic pump is 19 × 13 × 12 in., the capacitor unit is 22 × 15 × 10 in., and the cell is one-in. standard pipe of number 347 stainless-steel, 10-in. long. The equipment operates on 230 volts at 30 amperes.

Measurement of Insulating Quality of Textiles.—Central Scientific Company has announced the discovery of an application in the textile industry for one of its established products, a measuring apparatus for heat conductivity originally designed for other industrial and laboratory uses.

The insulating quality of blankets, overcoats and other fabrics also can be

tested accurately and quickly by the instrument, which until recently was thought to be capable of measuring the thermal conductivity of only such materials as blotting paper, wallboard, glass and all types of insulating materials, John T. Gossett, president, said. Textile manufacturers, he explained, are thus enabled to compare the warmth-retention qualities of various fabrics and to maintain specific standards before distribution to the garment industries.

According to Gossett, a layer of the material being tested is brought in contact with the measuring device which is filled with boiling water and kept at a constant temperature by an immersion heater. The material is then placed on a receiving block of metal of known heat capacity. The temperature changes in the receiver, as affected by the heat being conducted through the material, are measured by means of a thermocouple and the rate at which the receiver increases its temperature determines the insulating value of the fabric.

Well-insulated materials will cause the receiver block to warm up more slowly than those which are poorly insulated, Gossett explained. He also pointed out that the device is useful for measuring the lack of thermal conductivity, providing textile manufacturers with a positive means of determining the most comfortable fabrics for summer as well as winter.

The instrument, he said, is simple to operate and non-destructive to the material being tested.

Department stores are also reported to be interested in the apparatus. At the present time, Gossett said, Marshal Field & Company is employing the instrument in its Chicago testing laboratory for comparing the heat conductivity of synthetic fabrics with those made of wool.

Corrosion of in-storage ball bearings having brass retainers has been investigated by the Naval Research Laboratory, ONR, and found to be galvanic in nature. Corrosion rate is accelerated by small quantities of acids, formed by oxidation of petroleum or diesters, or by hydrolysis of an aliphatic diester oil, can be greatly reduced by using corrosion inhibitors capable of neutralizing acids as they are formed. High molecular weight, tertiary amine complexes of oil soluble organic acids have been found useful.

New Unit Provides Simulation of Backlash, Dead-Zone and Limit-Stops For Any Computer.—Computations involving Backlash, Dead-Zone and Limit-Stops now may be easily and accurately accomplished on any real-time analog computer such as IDA, REAC, or GEDA. Generation of these three non-linear functions is made possible by a new, compact unit, designated Model NLU-2, manufactured by the Computer Corporation of America, makers of "IDA."

The NLU-2 has two completely independent channels, each of which may be set for any of the three modes of operation. Variables of any magnitude normally encountered in commercial computers may be used as inputs, while complete and continuous front-panel adjustment of the function to be generated is quickly accomplished.

All-electronic in design, the NLU-2 may be used at higher frequencies than a servo-operated function generator. Although the accuracies of both types are comparable, the cost of obtaining backlash or dead-zone simulation on the two-channel NLU-2 is a small fraction of the cost of a single servo unit.

The sloping-front cabinet housing the NLU-2 measures 14 in. wide × 8 in. deep × 8 in. high. It has a self-contained power supply operating from a 115 volt, 60 cycle line and drawing ½ ampere.

An instrument for measuring horizontal deviation has been perfected at the Naval Ordnance Laboratory, White Oak, Md. Titled a "Horizontometer," the device has great sensitivity for small angular changes, yet will record deviation as great as 1800 degrees, or several complete turns. It consists of a float assembly fixed to the shaft of a potentiometer housing held in a container partially filled with fluid; the action of the resulting variable restoring buoyant forces on a lever arm is considered far superior to the old pendulum type of indicator with its low initial restoring torque.

Multiwall Paper Bags for Packaging Asphalt.—The Engineer Research and Development Laboratories, Fort Belvoir, Va., are investigating the practicability of packaging certain types of asphalt in multiwall paper bags for shipment to Army construction engineers in all parts of the world. The bags, currently undergoing rigid tests at ERDL's Packaging Development Laboratory, hold one hundred pounds of asphalt.

The asphalt is loaded into the bags in liquid form. This is done by heating it to pouring consistency. The liquid is then poured through a hose into the bags, which are placed in wooden scaffolds to help them retain their shapes. A clay coating on the inner bag liner prevents the hot asphalt from sticking to the paper and facilitates stripping of the bag from the contents.

After the bags are filled, they are stored for approximately 24 hours during which time the asphalt cools and hardens. This cooling and setting period is necessary before the packaged asphalt can be handled. Upon arrival at its destination, the asphalt is ready for use by merely cutting the bags with a sharp knife and stripping the paper.

Exposure, handling and storage tests in temperatures as low as 65 degrees below zero and as high as 165 above have been conducted at the Engineer Research and Development Laboratories. Field tests, including test shipments to the Canal Zone, are also being made.

Tests show that the bags, though deficient in some respects, nevertheless show some promise of meeting stiff military requirements. ERDL technicians are now working to overcome these deficiencies.

If the bags can be improved to the extent that they meet military requirements in all respects, they will be acceptable as alternates to the standard metal drums as a packaging medium.